The analytical solution of the problems in pipes measurements with IOS coordinate measuring machines

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Abstract: *The paper presents the analytical solution of determination of coordinate reference system for pipes system measurements. The transformation matrix is a composition of the consecutive calculated submatrices, basing on the chosen three points from frame of pipes.*

Keywords: pipes, measurements, transformation, matrices

1. Introduction

The pipes systems create the typical hydraulic parts in the motors construction. Therefore they are very commonly present in quality inspection in motor industry. The pipes systems are present in other disciplines of manufacturing of installations having similar form. These industrial installations consist of the connected pipes, creating rigid body to be inspected. The main strategy of measurements is to decompose this rigid body to the separately probed segments of known types. They can be approximated by the regular geometric elements or more complex shapes, basing on the sampled points data set. Checking of the dimensional requirements stated in technical documentation with regard to them can arise many problems. Existed differences of measured real shape from its ideal geometric form lead to the observation, that significant deviations from position and deformations of machined surfaces cannot supply the accurate analytical and at the same time useful results.

On the one hand the measured elementary surfaces with their estimated properties are representative ones for the standard coordinate measuring machine software tasks. On the other hand, due to their special properties, the particular methods of the measurement are suggested.

2. Subject

The main specified dimensional conditions are formulated to the locations of the differentiated sections of the whole pipes system. Especially important are the locations of the joint points coordinates, given by the certain number of nominal points coordinates with respect to the local coordinate system (the documentation or CAD model).

The paper considers the problem of construction of the optimal coordinate system of the pipes system, that is necessary to obtain the real values of specified locations of pipes connection points.

3. Method

The universal way in pipes measurements, used in IOS coordinate measuring machines, relies on two steps. The first one consists of building of the real rigid frame of pipes system. It is conducted by segmentation of the pipes system into single elementary geometric surfaces followed by probing of the necessary number of points, belonging to them (see Fig. 1). Each subset of points can determine the position, orientation and form parameters of the segmented surfaces. The obtained, after calculations, set of coordinates of joint points has reference to the global, machine coordinate system and describes the real rigid frame.

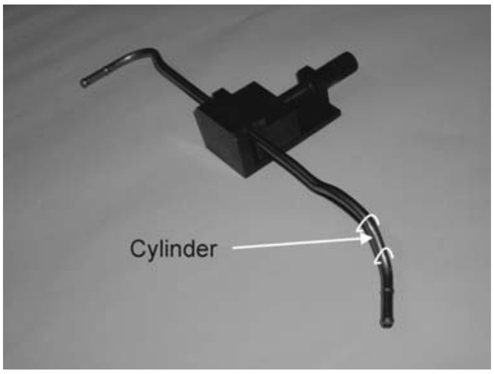

Fig. 1. The pipes system segmentation

The second step depends on fitting of this real frame, that should be approximately near to the nominal frame, specified by the default coordinates in the local coordinate system. After this operation, the previous calculated location points and estimated parameters are updated with regard to the new reference.

This transformation of real frame into nominal frame is the crucial computational alingment problem of pipes measurements, having the fundamental meaning on achieved results.

4. Solution

Geometric elements

Geometric elements, most frequently used in building of the pipes system model, are cylinders, tori, ellipses and cones. Estimation of their parameters is related to the "true distance", representing the perpendicular distance from measured point to the best-fit form element, modelling the real surface. Software implemented in IOS coordinate measuring machines is based upon the Gauss-Newton algorithm, which minimise the sum of squares of these distances d_i, according to the basic function in general form:

$$Q = \sum_{i=1}^{n} d_i^2(x, p)$$

Where

- x data points,
- p parameters,
- n number of measured points.

The minimum condition leads to nonlinear equations system in implicit form obtained by partially differentiating with respect to parameters p of the basic function. The solution for values of parameters, satisfying the above condition, is achieved on iterative way .

Building of real pipes frame

The straight sections of pipes system are usually approximated by cylinder models, however the curved pipe elements are modelled by torus surfaces. The parameters of position and orientation for cylinder axes and position for torus locations are suitable to determine of joint points of real frame. Concerning, that the real position of individual approximated elements does not fulfil the theoretic one, the intersection points of cylinder axes and points of tori locations have to be linked, using approximate adjustment.

5. Analytic transformation of the coordinate system

It should be noticed, that the real pipes frame is unambiguously placed in three dimensional Euclidean space, but the coordinates of joint points are associated with global coordinate system, i.e. machine coordinate system. The general purpose of pipes measurements is to obtain these values but related to the local frame system, in which the default set of nominal points coordinates are specified. The problem can be stated as finding the best values of factors of composite matrix that defines position and orientation of the global frame system with respect to local frame system. Solution of this problem goes through analytic transformation of one system to the second one, using three corresponding points (P_1, P_2, P_3), chosen appropriately or arbitrarily from frame body. The matrix will transform each of vertices of one triangle to another in such a way, that the distances between corresponding vertices will became as near as possible. The complete transformation is realised by the sequence of independent operations, that bring global system closer to local.

Translation

The first step is finding the submatrix of translation $\mathbf{T}_{TRANS}$, specifying translation factors (t_x, t_y, t_z) in *X*, *Y*, *Z* axes, respectively. Three fixed points create known but different triangles: the real and the nominal, oblique one. They ought to be identical, but in practice the observed distances between triangle vertices differ from fixed values. To settle this problem the centre points for both triangles are calculated. The components of $\mathbf{T}_{TRANS}$ matrix describe the displacement of one centre point to coincide with another and can be computed by simply subtracting coordinates of centres. The origin of system is placed into a centre point of oblique triangle, such that the both triangles have to be modified.

Rotation

After last transformation has been carried out, the both triangles have the same origin at their centre points, but they are still placed in different planes. The next task is then bringing the vector, perpendicular to the modified real triangle, to cover with the normal vector of the modified oblique triangle. If we denote the following vectors for both triangles:

$$^1\vec{Q} = {}^1\vec{N} \times {}^1\vec{S},$$
$$^2\vec{Q} = {}^2\vec{N} \times {}^2\vec{S}$$

Where

$$^1\vec{N} = \overrightarrow{({}^1P_1 - {}^1P_3)} \times \overrightarrow{({}^1P_1 - {}^1P_2)},$$
$$^2\vec{N} = \overrightarrow{({}^2P_1 - {}^2P_3)} \times \overrightarrow{({}^2P_1 - {}^2P_2)},$$

$$^1\vec{S} = \overrightarrow{({}^1P_3 - {}^1P_1)},$$
$$^2\vec{S} = \overrightarrow{({}^2P_3 - {}^2P_1)},$$

subscripts (^{1}) and (^{2}) denote the first and second triangle, respectively,

the matrix of rotation can be written as:

$$\mathbf{T}_{ROT} = {}^1\mathbf{T}_R^{\,T}\,{}^2\mathbf{T}_R$$

Where

$$^1\mathbf{T}_R^{\,T} = \begin{bmatrix} ^1S_x & ^1S_y & ^1S_z \\ ^1Q_x & ^1Q_y & ^1Q_z \\ ^1N_x & ^1N_y & ^1N_z \end{bmatrix},$$

$$^2\mathbf{T}_R = \begin{bmatrix} ^2S_x & ^2S_y & ^2S_z \\ ^2Q_x & ^2Q_y & ^2Q_z \\ ^2N_x & ^2N_y & ^2N_z \end{bmatrix}.$$

Alingment in designated plane

If the triangles are already placed in common plane the next task is to determine the adequate orientation angle for two remained axes. The angle can be expressed as a function of the distances between corresponding vertices of both triangles. In order to get the value of this angle the minimum condition of sum of distances squares is formulated. The estimated angular movement of axes can be expressed in the form of submatrix $\mathbf{T}_{ALING}$.

Scaling

The objective of scaling operation is again reducing the distances from vertices of nominal triangle to successive transformed triangle, which allows to compensate the differences between size of triangles. Also in this case, the criterion function, expressed by a sum of distances, is minimised to find of the optimal scaling factor. The submatrix is as follows:

$$\mathbf{T}_{SCALE} = \begin{bmatrix} t_S & 0 & 0 \\ 0 & t_S & 0 \\ 0 & 0 & t_S \end{bmatrix}$$

Where

t_s a scaling factor.

The transformation matrix

The final transformation matrix $\mathbf{T}$ can be composed of the successive calculated submatrices:

$$\mathbf{T} = \mathbf{T}_{SCALE}\,\mathbf{T}_{ALING}\,\mathbf{T}_{ROT}\,\mathbf{T}_{TRANS}$$

6. Results and Conclusion

The foregoing method of pipes measurements has been verified on IOS measuring machine (see Fig. 2). Performed tests on simulated data and practical measurements show the results are affected mostly from the choice of tree fixed points. In our method the suitable hierarchy of joint points is established. For inspected pipes these three points are the points, in which the hydraulic installation is to be maintained in motor construction, during assembly. This arrangement gives the satisfactory results however such particular strategy cannot be used universally. It's obvious that disadvantageous selection of basic triangle can provide the discrepancies between achieved results over accepted limit. For these cases the more general approach is desired. Analysing of this problem comes to the alternative method for estimating of the reference system matrix respecting all specified and observed joint points of pipes frame simultaneously. The further researches will allow to recognise these aspects of pipes measurements.

Fig. 2. NMP Linea MP_E =3.5+l/250 μm.

7. References

[1] Challis J. H.: A procedure for determining rigid body transformation parameters. *J. Biomechanics*, 1995, 28 (6): 733-737.

[2] Press W. H., Teukolsky S. A., Vetterling W. T.: Numerical Recipes in C. *The Art. Of Scientific Computing*, Cambridge University Press, 1995.