The Design of Continuous Scale Calibration Experiment

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Abstract. *This paper deals with evaluation of continuous scale calibration experiments. Especially shows the designs of experiments which we are able to evaluate.*

Keywords: Continuous Scale Calibration, Calibration, Metrology, Measurement, Measuring device

Introduction

It is serious problem to create the design of calibration experiment at the calibration of continuous scale measuring devices so, that the parameters of calibration function should be estimated. It relates with that we have to be able to estimate the parameters of second grade in calibration model. Part of that second grade parameters we estimate by the type B method (type B uncertainties) at the base of previous experiments information; part is estimated by the type A method (type A uncertainties) from present experiment data. Consider the model of calibration evaluating

$$E(\boldsymbol{W}) = \boldsymbol{A\beta} \tag{1}$$

where E($\boldsymbol{W}$) is expectation of random vector $\boldsymbol{W}$, $\boldsymbol{W}$ n-dimensional observation vector, A known matrix of type $n \times p$, $\boldsymbol{\beta}$ is p-dimensional vector of unknown parameters. Excessively often for our purposes

$$\boldsymbol{W} = \boldsymbol{Y}_1 + \boldsymbol{C}_1\boldsymbol{Y}_2 + \boldsymbol{D}(\boldsymbol{X}_1 + \boldsymbol{C}_2\boldsymbol{X}_2) = (\boldsymbol{I} \quad \boldsymbol{C}_1 \quad \boldsymbol{D} \quad \boldsymbol{DC}_2)\begin{pmatrix}\boldsymbol{Y}_1\\ \boldsymbol{Y}_2\\ \boldsymbol{X}_1\\ \boldsymbol{X}_2\end{pmatrix} \tag{2}$$

where $\boldsymbol{I}$ represents the identity matrix, $\boldsymbol{Y}_1$ vector of quantity values measured by measuring standard, $\boldsymbol{Y}_2$ vector of all influences affecting at realization of measurement by measuring standard, $\boldsymbol{X}_1$ vector of quantity values measured with calibrated measuring device, $\boldsymbol{X}_2$ vector of all influences affecting at realization of measurement with calibrated measuring device, $\boldsymbol{C}_1$, $\boldsymbol{C}_2$, $\boldsymbol{D}$ are known matrixes which determines distribution of influent quantities and errors in model. Then the covariance matrix of random vector $\boldsymbol{W}$ (for dependence only between $\boldsymbol{X}_1$ and $\boldsymbol{Y}_1$ and also only between $\boldsymbol{X}_2$ and $\boldsymbol{Y}_2$) will be

$$\boldsymbol{U}_W = (\boldsymbol{I} \quad \boldsymbol{C}_1 \quad \boldsymbol{D} \quad \boldsymbol{DC}_2)\begin{pmatrix}\boldsymbol{U}_{Y_1} & \boldsymbol{0} & \boldsymbol{U}_{Y_1,X_1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{U}_{Y_2} & \boldsymbol{0} & \boldsymbol{U}_{Y_2,X_2}\\ \boldsymbol{U}_{X_1,Y_1} & \boldsymbol{0} & \boldsymbol{U}_{X_1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{U}_{X_2,Y_2} & \boldsymbol{0} & \boldsymbol{U}_{X_2}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}\\ \boldsymbol{C}_1'\\ \boldsymbol{D}'\\ \boldsymbol{C}_2'\boldsymbol{D}'\end{pmatrix} =$$

$$= \boldsymbol{U}_{Y_1} + \boldsymbol{C}_1\boldsymbol{U}_{Y_2}\boldsymbol{C}_1' + \boldsymbol{DU}_{X_1}\boldsymbol{D}' + \boldsymbol{DC}_2\boldsymbol{U}_{X_2}\boldsymbol{C}_2'\boldsymbol{D}' + \boldsymbol{DU}_{X_1,Y_1} +$$

$$+\boldsymbol{U}_{Y_1,X_1}\boldsymbol{D}'+\boldsymbol{D}\boldsymbol{C}_2\boldsymbol{U}_{X_2,Y_2}\boldsymbol{C}_1'+\boldsymbol{C}_1\boldsymbol{U}_{Y_2,X_2}\boldsymbol{C}_2'\boldsymbol{D}'=$$
$$=\sigma_Y^2\boldsymbol{H}_{Y_1}+u_Y^2\boldsymbol{C}_1\boldsymbol{H}_{Y_2}\boldsymbol{C}_1'+\sigma_X^2\boldsymbol{D}\boldsymbol{H}_{X_1}\boldsymbol{D}'+u_X^2\boldsymbol{D}\boldsymbol{C}_2\boldsymbol{H}_{X_2}\boldsymbol{C}_2'\boldsymbol{D}'+$$
$$+\sigma_{X,Y}(\boldsymbol{D}\boldsymbol{H}_{X_1,Y_1}+\boldsymbol{H}_{Y_1,X_1}\boldsymbol{D}')+u_{X,Y}(\boldsymbol{D}\boldsymbol{C}_2\boldsymbol{H}_{X_2,Y_2}\boldsymbol{C}_1'+\boldsymbol{C}_1\boldsymbol{H}_{Y_2,X_2}\boldsymbol{C}_2'\boldsymbol{D}') \quad (3)$$

where $\boldsymbol{A}'$ represents transposition of matrix $\boldsymbol{A}$ segments, $\sigma_Y^2\boldsymbol{H}_{Y_1}$ is the type A covariance matrix for measurements made by standard, $u_Y^2\boldsymbol{H}_{Y_2}$ is the type B covariance matrix for measurements made by standard, $\sigma_X^2\boldsymbol{H}_{X_1}$ is type A covariance matrix for measurements made by calibrated measuring device, $u_X^2\boldsymbol{H}_{X_2}$ is the type B covariance matrix for measurements made by calibrated measuring device, $\sigma_{X,Y}\boldsymbol{H}_{X_1,Y_1}$ is the type A covariance matrix measurements made by standard and a calibrated measuring device, for which is given $\sigma_{X,Y}\boldsymbol{H}_{X_1,Y_1}=\sigma_{X,Y}\boldsymbol{H}'_{Y_1,X_1}$, $u_{Y,X}\boldsymbol{H}_{X_2,Y_2}$ is the type B covariance matrix for measurements made by standard and a calibrated measuring device, for which is given $u_{Y,X}\boldsymbol{H}_{X_2,Y_2}=u_{Y,X}\boldsymbol{H}'_{Y_2,X_2}$. We assume known uncertainties and covariances of type B and knowledge of weights (matrixes ***H***) between uncertainties or covariances of type A but not for parameters σ_X^2 , σ_Y^2, $\sigma_{X,Y}$, in covariance matrix (3) .

For entry in form

$$\boldsymbol{W}=\boldsymbol{Y}_1+\boldsymbol{D}\boldsymbol{X}_1+\boldsymbol{C}_1\boldsymbol{Y}_2+\boldsymbol{D}\boldsymbol{C}_2\boldsymbol{X}_2=(\boldsymbol{I},\boldsymbol{D})(\boldsymbol{Y}_1,\boldsymbol{X}_1)'+(\boldsymbol{C}_1,\boldsymbol{D}\boldsymbol{C}_2)(\boldsymbol{Y}_2,\boldsymbol{X}_2)'=\boldsymbol{Z}_1+\boldsymbol{C}\boldsymbol{Z}_2 \quad (4)$$

and covariance matrix for independence between $\boldsymbol{Z}_1$ and $\boldsymbol{Z}_2$ will be

$$\boldsymbol{U}_W=(\boldsymbol{I},\boldsymbol{D})\begin{pmatrix}\boldsymbol{U}_{Y_1} & \boldsymbol{U}_{Y_1,X_1}\\ \boldsymbol{U}_{X_1,Y_1} & \boldsymbol{U}_{X_1}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}\\ \boldsymbol{D}'\end{pmatrix}+(\boldsymbol{C}_1,\boldsymbol{D}\boldsymbol{C}_2)\begin{pmatrix}\boldsymbol{U}_{Y_2} & \boldsymbol{U}_{Y_2,X_2}\\ \boldsymbol{U}_{X_2,Y_2} & \boldsymbol{U}_{X_2}\end{pmatrix}\begin{pmatrix}\boldsymbol{C}_1\\ \boldsymbol{C}_2'\boldsymbol{D}'\end{pmatrix}=\boldsymbol{U}_{Z_1}+\boldsymbol{C}\boldsymbol{U}_{Z_2}\boldsymbol{C}' \quad (5)$$

about $\boldsymbol{U}_{Z_1}$ we assume that is in the form

$$\boldsymbol{U}_{Z_1}=(\boldsymbol{I},\boldsymbol{D})\begin{pmatrix}\sigma_Y^2\boldsymbol{H}_{Y_1} & \sigma_{X,Y}\boldsymbol{H}_{Y_1,X_1}\\ \sigma_{X,Y}\boldsymbol{H}_{X_1,Y_1} & \sigma_X^2\boldsymbol{H}_{X_1}\end{pmatrix}\begin{pmatrix}\boldsymbol{I}\\ \boldsymbol{D}'\end{pmatrix}$$

where $\boldsymbol{I}$ is identity matrix, $\boldsymbol{U}_{Z_1}$ is the type A covariance matrix and $\boldsymbol{U}_{Z_2}$ is the type B covariance matrix. Covariance matrix (5) is known, but not for the second grade parameters σ_X^2 , σ_Y^2, $\sigma_{X,Y}$. We can't find estimates of these parameters in generally. We will show in which calibration experiments it is posible, in the next.

The Evaluation of Continuous Scale Calibration Model

For needs in calibration evaluating by continuous scale measuring devices in practice at nowadays are using folowing cases of calibration model.

1) We know covariance matrix $\boldsymbol{U}_W$ (3) of observation vector ***W***
2) We know weight matrix ***H*** of observation vector ***W*** for which is given $\boldsymbol{U}_W=\sigma^2\boldsymbol{H}$ and σ^2 we don't need to know
3) In special case if observation vector ***W*** is in form (4), and exists such a matrix ***Q***, that

$$\boldsymbol{C}=\boldsymbol{A}\boldsymbol{Q} \quad (6)$$

where ***A*** is the design matrix of calibration model (1) and ***C*** is known matrix in relation (4).

We can evaluate this cases (see [1], [2], [3]).

The Design of Continuous Scale Calibration Experiment

We are not able to evaluate the calibration experiment always. We can evaluate the calibration in three cases stated in chapter 2. The mathematical statistics can offer another solutions, which should be suitable for continuous scale calibration, besides that. Inasmuch as in metrological practice are not always use the modern results of statistical theory enaugh, we show here possible designs of calibration experiment, which is possible to evaluate at the base of mathemathical statistic results stated in chapter 2. The most frequently are calibration experiment models in generally dividing in to :

B) Models with replication of measurement in values of measured quantity (in measured points). This case is posible to evaluate if at least 10 replication of measurement in values of measured quantity are done. We use standard statistical estimates for estimation of σ_X^2 , σ_Y^2, $\sigma_{X,Y}$ from replication of measurement, see e.g. [1] pg.31 and pg.115.

B) Models without replication of measurement in values of measured quantity (in measured points).Consider these folowing situations for this case

1) The type B uncertainties and covariances have no influence on estimate of model parameters (1. and 2. grade)
2) The type B uncertainties and covariances have influence on estimate of model parameters (1. and 2. grade)

Case B.1.) denotes that is sufficient to know part of covariance matrix (3) for calibration model parameters estimates

$$\boldsymbol{U}_W = \sigma_Y^2 \boldsymbol{H}_{Y_1} + \sigma_X^2 \boldsymbol{D}\boldsymbol{H}_{X_1}\boldsymbol{D}' + \sigma_{X,Y}\boldsymbol{H}_{X_1,Y_1} \tag{7}$$

Case B.1.) we may have consider when the type B uncertainties and type B covariances are negligible owe to the type A uncertainties and covariances. This case arise too, when for observation vector $\boldsymbol{W}$ in form (4) is fulfilled the condition (6).

For case B.1.) may have arise these situations

a) The type A uncertainty of standard measuring device is zero (e.g. standard is mass measure), the type A covariance is zero (in this case this condition flows from previous condition). We can evaluate this calibration design and it answer to 2.case calibration evaluating from chapter 2.

b) The type A covariance is zero and we know rate between the type A uncertainties of calibrated measuring device and standard $k = \frac{\sigma_X}{\sigma_Y}$. We can evaluate this calibration design and it answer to 2.case calibration evaluating from chapter 2.

c) The type A covariance is zero and we know that the matrixes of weights for standard and for calibrated measuring device are the same, so $\boldsymbol{H}_{Y_1} = \boldsymbol{D}\boldsymbol{H}_{X_1}\boldsymbol{D}'$. We can evaluate this calibration design and it answer to 2.case calibration evaluating from chapter 2.

d) We can evaluate calibration experiment only in a), b), c) cases, for case B.1.). We can evaluate another cases only if we replicate the measurement, for case B.1.).

Case B.2.) denotes that we will consider covariance matrix in form (3) or (5) for calibration model parameters estimates. We assume that in matrix (3) we know all matrixes and parameters without parameters σ_X^2 , σ_Y^2 and $\sigma_{X,Y}$ which denotes the parameters of uncertainties earned by the type A method (by statistical analysis) from present calibration experiment data.

For case B.2) may have arise these situations

a) We know the type B and type A uncertainties rates for standard and calibrated measuring device. We know type B covariance to type A covariance rate between standard and calibrated measuring device. In this are known rates

$$k_Y = \frac{u_Y^2}{\sigma_Y^2}, k_x = \frac{u_X^2}{\sigma_X^2}, k_{X,Y} = \frac{u_{X,Y}}{\sigma_{X,Y}}$$

We can evaluate this calibration design and it answer to 2.case calibration evaluating from chapter 2.

Especially if the type B uncertainty of calibrated measuring device is zero, what arise in cases when data of calibrated measuring device was observed directly from calibrated measuring device then will be $k_x = 0$ and it is necessary to know rate between the type A uncertainties of standard and calibrated measuring device.

This case may have arise even in situation that we know in place of the rate $u_{X,Y} / \sigma_{X,Y}$, the type A covariance to type A uncertainties rate of standard or calibrated measuring device $\sigma_{X,Y} / \sigma_Y^2$, or $\sigma_{X,Y} / \sigma_X^2$.

Special case is when the type A covariance is zero ($\sigma_{X,Y} \boldsymbol{H}_{X_1,Y_1} = 0$). We can evaluate this calibration design and it answer to 2.case calibration evaluating from chapter 2.

b) We can evaluate calibration experiment only in a) case, for case B.2.). We can evaluate another cases only if we replicate the measurement, for case B.2.).

Conclusion

In paper are described the designs of calibration experiment suitable for the calibration evaluating. The calibration we can always evaluate when measurements are adequately replicated in calibration points. Other situation arises when we can't replicate the measurements or the replication is financially demanding. When we don't replicate the measurement and uncertainties of type B has not influence on the estimates of model parameters, is sufficient if the type A uncertainty of standard is zero or if we know the rate between the type A uncertainty of calibrated measuring device and standard, and we assume the type A covariance between standard and calibrated measuring device is zero in both cases, then we can evaluate the calibration experiment. If the type A covariance between standard and calibrated measuring device are not zeros we need to know their rate to uncertainty of standard or calibrated measuring device. In case when we don't replicate the measurements and the type B uncertainty has influence on the calibration function estimates, we can evaluate when we know the rates between the type B uncertainty and the type A uncertainty for standard and calibrated measuring device, and rate between the type B and type A covariances between standard and calibrated measuring device. If we would not knew the rate between the type B and type A covariances between standard and calibrated measuring device then is sufficient if we would knew rate of the type A covariance to the type A uncertainty of standard or rate to the type A uncertainty of calibrated measuring device. For all marked cases we assume known measurement weights and relations between the type A and type B uncertainties in different calibration points and relations between mutual covariances, too.

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