Comparison of Self-Calibration Methods for Measurement Channels in Respect of a Method of Conversion Functions Interpolation

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Abstract: *The paper suggests algorithms enabling self-calibration of measurement channels with linear as well as nonlinear conversion functions. To approximate conversion characteristics interpolation methods are used. They are widely known, however the essence of the suggested self-calibration method is current modernization of interpolating functions parameters due to suitable procedure. It enables correction of the two essential error components, which may appear in the measurement channel, by means of evaluating of the current value of the real slope of the conversion characteristic as well as of the components describing nonlinearity generated (or modified) by disturbing factors. In the paper methods of self-calibration for measurement channels cooperating with non-electrical quantities sensors have been compared. Interpolation of conversion functions has been assumed as linear in intervals and polynomial.*

Keywords: self-calibration, non-electrical monitoring, sensors, interpolation.

1. Introduction

Measuring systems operating during prolonged measurements as well as in terms of significant scattering of measurement points require specific procedures of self-diagnostics and analysis of technical state for individual elements of a measurement channel. Likewise an analysis of methods of self-calibration of the system measurement channels is necessary, independently on initial calibration of the measurement channel. They make possible rendering independent measurements results from possible changes of electronic elements parameters influence. Procedures of self-calibration most frequently refer to accomplishment of prolonged measurements (monitoring) of non-electrical quantities. They enable also: either to avoid troublesome initial adjustment of the measurement channels elements, or to use cheaper elements of lower quality, without worsening of the accuracy and stability of the whole measurement system [1]. Self-calibration procedures are mostly limited to electronic parts of measurement systems and consist in current evaluation of present values of the conversion functions parameters. It is necessary to employ calibrating reference elements (depending on the sensor type, e.g. standard voltage sources, standard resistors, voltage dividers etc.). Two elements are necessary for linear conversion function, more for non-linear one [2, 3]. For emphatic majority of cases, self-calibration procedures do not include sensors, because it would require troublesome and frequently difficult in realization using of measured non-electrical quantities standards.

2. Measurement Channel Sensitivity

Conversion function of a channel for measurement of a non-electrical quantity (Fig.1) may be expressed as a composition of a sensor conversion function f and electronic part of the measurement channel g [2]. For general case of non-linear functions f and g:

$$y = f(x), \tag{1}$$

$$z = g(y, a_0, a_1, \dots a_n), \tag{2}$$

where: $a_0,a_1,\dots a_n$ – parameters of the conversion channel of the measuring apparatus, which may change due to time, temperature, or other influencing factors. Substituting (1) into (2) we have:

$$z = g\big(f(x), a_0, a_1, \dots a_n\big). \tag{3}$$

Assuming, that sensitivities of the measuring apparatus and of the sensor are respectively S_A and S_s, the measurement channel sensitivity is:

$$S = \frac{dz}{dx} = \frac{\partial z}{\partial y} \cdot \frac{dy}{dx} = g'(y) \cdot f'(x) = S_A(y, a_0, a_1, \dots a_n) \cdot S_S(x). \tag{4}$$

The resultant sensitivity of a measurement channel is a product of sensitivities of all its elements. Such approach makes possible to mould the total measurement channel sensitivity by means of the choice of the individual elements sensitivities.

Let's consider a case, when the sensor conversion characteristic is either linear, or non-linear, but known and invariable, but the parameters of the channel conversion function may vary with time or disturbing factors during prolonged measurement. Let's assume additionally, that the conversion function of the apparatus $S_A(y,a_0,a_1\ldots a_n)$ may be in general case non-linear. As a matter of fact, at the stage of the apparatus manufacturing it is possible to fit its parameters in the way eliminating non-linearity errors, but during exploitation, particularly within prolonged working cycle and with significant influence of disturbing factors, it is necessary to take into consideration possibilities of the conversion function parameters changes and appearing of non-linearity errors. By means of multipoint calibration [4,5] (Fig.2.) it is possible to evaluate present values of the conversion function parameters. The results of measurements between calibration points are evaluated by means of interpolation: linear, linear in intervals, or with polynomial functions, when interpolation knots are determined by the values of calibrating quantities $y_0,y_1\ldots y_n$, substituting during calibrating procedure the output signal y of the sensor. Dependently on the type of the sensor output quantity, as the reference quantities voltage sources, standard resistors, voltage dividers etc. are accepted. Position "y" of the switch P denotes function of measurement of the unknown non-electrical quantity x.

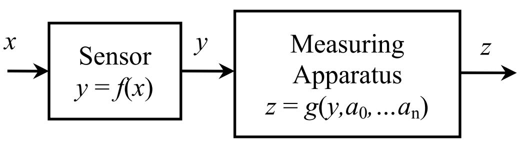

Fig. 1. Measurement channel diagram.

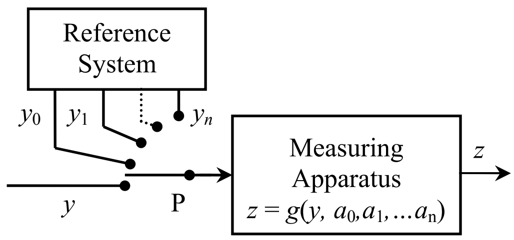

Fig. 2. Schematic diagram of a measurement channel self-calibration.

3. Algorithm of a Channel Self-Calibration by Means of Linear in Intervals Interpolation of the Conversion Function

Let's present interpolation of a conversion function of a measuring apparatus with spline functions of 1st order (so called functions linear in intervals) in closed interval <α; β> for $(n+1)$ calibration points $y_0,y_1,\ldots y_n$, while:

$$\alpha = y_0 < y_1 < \ldots < y_{n-1} < y_n = \beta \,. \tag{5}$$

Points y_i for $i = 0,1, \ldots n$ ($n \geq 1$) determine a partition of the interval <α; β> to n subintervals, and assuming that this partition is uniform, we have:

$$y_i = y_0 + i \cdot h \qquad h = \frac{\beta - \alpha}{n} \qquad i = 0,1,\ldots,n \,. \tag{6}$$

Let's represent linear in intervals interpolation $\varphi(y)$ of the conversion function of measurement apparatus $g(y,a_0,a_1\ldots a_n)$ (2):

$$\varphi_i(y) = \frac{1}{h} \cdot \left[(z_i - z_{i-1}) \cdot y + (z_{i-1}y_i - z_i y_{i-1})\right] \qquad i = 1,\ldots,n \,. \tag{7}$$

It is easy to notice, that at the interpolation knots, determining i-th interval $<y_{i-1}; y_i>$, the values of interpolating function are respectively:

$$\varphi_i(y_{i-1}) = g(y_{i-1}) = z_{i-1} \qquad \varphi_i(y_i) = g(y_i) = z_i \qquad i = 1,\ldots,n \,. \tag{8}$$

Using notations:

$$p_i = \frac{1}{h} \cdot (z_i - z_{i-1}) \qquad q_i = \frac{1}{h} \cdot (z_{i-1}y_i - z_i y_{i-1}) \qquad i = 1,\ldots,n \,, \tag{9}$$

we get the final relationship for the interpolating function within the i-th interval:

$$\varphi_i(y) = p_i \cdot y + q_i \qquad i = 1,\ldots,n \,. \tag{10}$$

For the assigned $n+1$ standard values $y_0,y_1,\ldots y_n$ (Fig.2.) it is necessary to determine $n+1$ corresponding output values $z_0, z_1,\ldots z_n$ of the equipment (according to (8)) and then to determine coefficients p_i and q_i of the interpolating function according to (9).

Application of this type interpolation is known, however the essence of the suggested method is the current modernization of the interpolating function parameters p and q due to application of a suitable self-calibration procedure. It is then possible to correct two essential error components (sensitivity and non-linearity errors), which may appear in the measurement channel, by means of evaluating of the current value of the conversion characteristic real slope and the components describing non-linearity generated (or modified) by disturbing factors in the measurement channel.

4. Self-Calibration Algorithm Using Polynomial Interpolation of the Conversion Function

Let's expand conversion function of the apparatus $g(y,a_0,a_1\ldots a_n)$ (2) into Taylor series according to the variable y powers within zero neighborhood ($a_0,a_1\ldots a_n$ are the function parameters):

$$z(y) = z(0) + \tfrac{1}{1!}\cdot y \cdot z'(0) + \tfrac{1}{2!}\cdot y^2 \cdot z''(0) + \ldots \tag{11}$$

Using notations:

$$a_0 = z(0), \quad a_1 = \tfrac{1}{1!}\cdot z'(0), \quad a_2 = \tfrac{1}{2!} z''(0), \ \ldots \quad \text{and} \quad z_0 = z(y_0), \quad z_1 = z(y_1), \quad z_2 = z(y_2), \ldots \tag{12}$$

and taking them into account in (11) we obtain for n+1 knots y_0, y_1, … y_n, for which the values of the interpolated function are equal $z_0,z_1,\ldots z_n$, a system of n+1 equations with searched coefficients $a_0,a_1,\ldots a_n$ of the conversion function. Using n+1 calibration points (n+1 knots) and accepting order n of the interpolating polynomial it is possible for arbitrary spacing of interpolation knots y_i to evaluate the Lagrange interpolating polynomial $\varphi(y)$:

$$\varphi(y) = \sum_{i=0}^{n} z_i \cdot \frac{\omega_n(y)}{(y-y_i)\cdot \omega_n'(y_i)}, \tag{13}$$

where:

$$\omega_n(y) = (y-y_0)(y-y_1)\ldots(y-y_n), \tag{14}$$

and $\omega_n'(y_i)$ is the value of the polynomial $\omega_n(y)$ derivative at the point y_i .

To approximate real conversion function (2) of a measurement apparatus with a polynomial of the n^{th} order it is necessary to assign n+1 standard values $y_0,y_1,\ldots y_n$ (Fig.2.) and to determine n+1 corresponding output values $z_0,z_1,\ldots z_n$. Interpolating polynomial, making possible to determine z value for arbitrary y from among y_i and y_j is determined on the grounds of (13) and (14).

The essence of the suggested method is the current modernization of the interpolating polynomial parameters due to application of a suitable self-calibration procedure (Fig.2.).

Similar effect of self-calibration may be achieved also with other interpolating functions. In any case it is necessary to use reference standard quantities (Fig.2). Their accuracy and stability will decide on the self-calibration procedure accuracy and efficiency.

5. Comparison of Self-Calibration Algorithms

The efficiency of the discussed self-calibration methods depends on the type of nonlinearity of the real conversion characteristic of measurement apparatus as well as on the number of the used interpolation knots (number of calibration points). The interpolation error for conversion characteristic is:

$$\Delta = S_A(y)\cdot y - \varphi(y). \tag{15}$$

As a comparison criterion we'll use limit error value:

$$\Delta_{\max} = \max|\Delta|. \tag{16}$$

For this criterion in the paper there have been compared self-calibration methods with linear in intervals and polynomial interpolation of conversion characteristics. The comparison has been made for exemplary, typical measurement channels, containing in the apparatus part converters: 'relative change of strain gauge bridge unbalance voltage' to frequency, whose conversion function is [2]:

$$z = z_0 + k_1 \cdot y + \frac{k_2}{1 + k_3 \cdot y}, \tag{17}$$

where: z – frequency, y – relative bridge unbalance voltage, $k_1 \div k_3$ – parameters of measurement apparatus, as well as a dual-slope integrating A/D converter with conversion function:

$$z = k \cdot \ln[1 + y \cdot (1 - \mathrm{e}^{-k})], \tag{18}$$

where: z – relative time (output time to integration time), y – relative voltage (output to reference voltage), k – aggregated parameter of A/D converter.

In these channels it has been taken into consideration the influence of the real values of electronic elements expressed by apparatus parameters k and $k_1 \div k_3$ and their possible changes caused by disturbances.

As an example the Fig.3 shows dependence of limit error (16) on the number of calibration points (interpolation knots) for the converter 'relative change of strain gauge bridge unbalance voltage' to frequency (17) taking into consideration real properties of amplifiers (offset causing nonlinearity). The Fig.4 shows dependence of the limit error (16) on the number of calibration points (interpolation knots) for the dual-slope A/D converter (18), taking into consideration dielectric loss of the integrating capacitor. The self-calibration methods used are: linear in intervals interpolation of conversion function (curve 1 on the figures) and polynomial interpolation of conversion function (curve 2 on the figures).

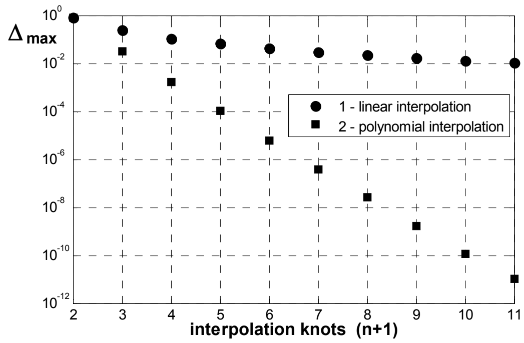

Fig.3. Limit error (16) versus the number of calibration points for the converter 'bridge unbalance voltage' to frequency.

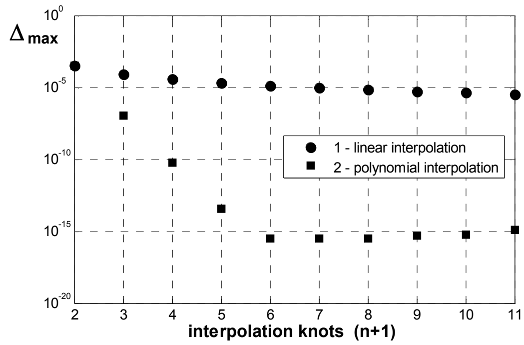

Fig.4. Limit error (16) versus the number of calibration points for the dual-slope A/D converter.

On the grounds of the diagrams it may be ascertained, that for the analyzed exemplary measurement channels the polynomial interpolation method gives quicker decrease of the limit error. For two interpolation knots (n=1) the effects of the methods efficiency agree, which is obvious, so the values of limit error (16) are identical. For three and larger number of interpolation knots the error of polynomial interpolation decreases rapidly, giving satisfactory values by three knots. In the case of interpolation with function linear in intervals the interpolation error decreases very slowly. For the example from the Fig.3 the error of linear interpolation decreases for two orders not before 11 interpolation knots, while for polynomial interpolation 3 knots are enough, as for the example from the Fig.4. Delimiting of further decrease of the error (Fig.4) to the value of 10^{-15} is the result of limited exactness of calculations performed in the Matlab environment. In the case of self-calibration procedure accomplishment the application of polynomial interpolation will make possible to delimit essentially the number of necessary reference sources (Fig.2).

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