Iterative method and averaging for measurement error correction

[1]M. Kamenský, [2]K. Kováč

Department of Measurement, Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Ilkovičova 3, 81219 Bratislava, Slovak Republic
Email: [1]miroslav.kamensky@stuba.sk, [2]karol.kovac@stuba.sk

Abstract. *Iterative method of measurement error correction is presented. The method is useful for digital measurements e.g. for successive approximation ADC. In this case method impact on accuracy could be enhanced using oversampling and averaging as proved on the basis of experimental results with evaluation of ENOB and SNR.*

Keywords: measurement error correction, iterative method, oversampling

1. Introduction

Accuracy is the dominant metrological property of a measurement unit including intelligent sensor systems. Some errors change the value in time because of component aging and time drift. Digital components used in today measurement devices bring new possibilities to error correction in real time. An algorithm implemented in a microcontroller and employing a few additional analog components could be used for an error correction. Digital measurement procedure could be described by PSIQ model as a product of discrete-mathematical approach [1]. Then structural-algorithmic methods used for digital measurement device correspond to model of second order [2]. In this paper iterative method of measurement accuracy improvement is described on the basis of experimental results.

2. Subject and Methods

Discrete-mathematical approach of measurement procedure has been described in [1]. In ratio scale the case of an adaptive partition has been described, which is suitable e.g. for successive approximation Analog to Digital Converter (ADC). The model is stated as

$$\min\left\{ \, |f^{-1}(b)-a| \le \varepsilon n \;\middle|\; b=f(a)\in \boldsymbol{B} \right\} \qquad (1)$$

where

$\boldsymbol{A}, \boldsymbol{B}, n$ — $\boldsymbol{A}\subset\boldsymbol{Z}^{+}$, $\boldsymbol{A}=\{0, 1, ..., a_n\}$, $|\boldsymbol{A}|=n$ - an empirical system – set of values of the physical quantity with a given unique order relation < over it; $\boldsymbol{B}=\{b_1, b_2, ..., a_n\}$, $\boldsymbol{B}\subset\{0,1\}^n$, $|\boldsymbol{B}|=n$ - a numerical system – set of numerical values with a given unique order relation < over it

$a, b, f(a)=b$ — $a\in\boldsymbol{A}$, $b\in\boldsymbol{B}$ element of the set; $f(a)=b$ homomorphism mapping a into b

ε — given relative error of measurement.

The term PSIQ-model is used for a four-unit framework, which describes most measurement procedures using four types of primary operations hidden behind the Eq. 1. These units are: the Partitioner (P), the Selector (S), the Inverter (I), the Questioner (Q). One way of measurement accuracy improvement is structural-algorithmic methods. An example is iterative method known also as the inverse transformation method. The measurement errors are diminished with auxiliary means formally described with model of second order SCI-model, see Fig. 1. [2]

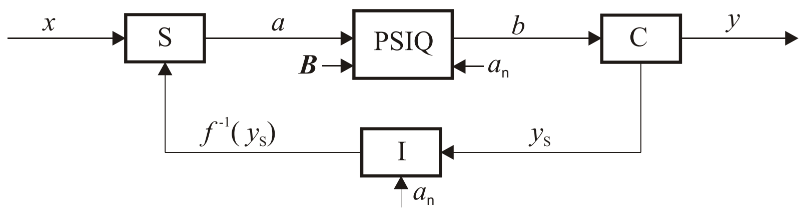

Fig. 1. SCI model of measurement. S-element – realizes a choice between inputs to PSIQ; C-element executes calculation and memorization; I-element fulfils inverse mapping of the result into the empirical set (with reference a_n); PSIQ-element is the corrected measurement device described by PSIQ model.

The iterative formula of the method is

$$y_{s,i} = y_{s,i-1} + (b_0 - b_i) \tag{2}$$

where

b_0, b_i the output of ADC in the initial step b_0, which corresponds to the measured value *x,* or in the next steps b_i corresponding to output of I-element

$y_{s,i}$ the actual corrected value in the step *i*, which is memorized and put into the input of I-element.

The condition of convergence should be fulfilled [2] and the appropriate ending condition of the process (occurrence of oscillation) has been designed in [3] with the formula for final output calculation. The method leads to suppression of nonlinearity, but is restricted by resolution of ADC. This could be overcome with help of averaging [4]. Under the assumption that the analog input signal is band-limited and the sampling frequency satisfies Nyquist theorem, the white noise model shows that the error average power is given by

$$\sigma^2_{e1D} = \frac{q^2}{12}\frac{1}{m} \tag{3}$$

where

m oversampling ratio $m = f_{s1}/f_N$, $f_{s1} > f_N$ is sampling and Nyquist frequency

q quantisation step given by resolution of ADC.

This formula suggests that the conversion accuracy can be improved by refining resolution not only in amplitude but also in time. Technique refining time discretization is referred to as oversampled analog-to-digital conversion. Better conversion accuracy with the same oversampling ratio *m* is attaining using nonlinear reconstruction algorithms as in [4]. Theory for other than white noise included in input signal is presented e.g. in [5]. Besides oversampling also decimation (averaging) should be performed to achieve better resolution with final frequency of sample results f_N. It could be shown, that oversampling improves Effective Number of Bits (*ENOB*) and Signal to Noise Ration (*SNR*)

$$ENOB = B + \log_2 \sqrt{m} \tag{4}$$

$$SNR = B\,20\log_{10} 2 + 10\log_{10} 12 + 10\log(m) \cong 6{,}02B + 1.76 + 10\log(m) \tag{5}$$

where *B* is nominal number of bits of ADC.

Measurements were made in static input voltages uniformly spread over the input range. This could be seen as a slowly input saw-tooth signal, for which the ENOB could be calculated as

$$ENOB = \frac{10\log\frac{\sigma_x^2}{\sigma_e^2}}{20\log_{10}2} = \frac{SNR}{20\log_{10}2} \tag{6}$$

where

σ_x, σ_e root mean square (rms) of input and output respectively, after oversampling and decimation is $\sigma_e = \sigma_{e1D}$.

3. Results

The testing workplace for the iterative method (Eq. 2) has been described in [3]. The method is suitable for error correction of analog input channel of cheap measuring devices, very often represented by ADC of the microcontroller. The I-element has been implemented by PWM output and a simple low-pass RC-filter. The parameters of the RC-filer should assure that the output fluctuations cause negligible error (0.1 LSB). Slow response of I-element output limits the sampling frequency ($f_s = 0.1$ Hz) of iterations maintaining algorithm convergence. 20 processes in each of 51 input levels equally spread through the whole input range have been measured and evaluated. Measurement errors curve before and after correction is depicted in the Fig. 2. In experiments the correction decreased total average error from 1.7 LSB to 0.7 LSB and maximal error from 2.1 LSB to 1.3 LSB. Tendency of correction error to error of PWM has not been proved in this case because of dominating quantisation error of ADC.

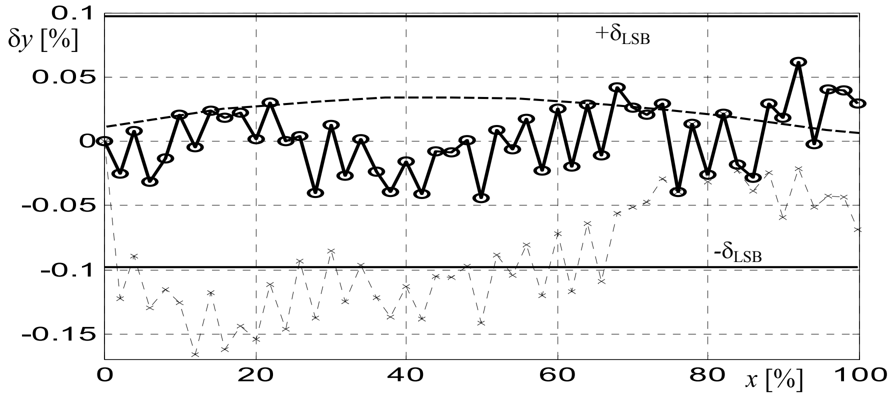

Fig. 2. Average measurement error curve before (dotted line with x marker) and after correction (solid line with o marker). The dashed line represents error of PWM circuits.

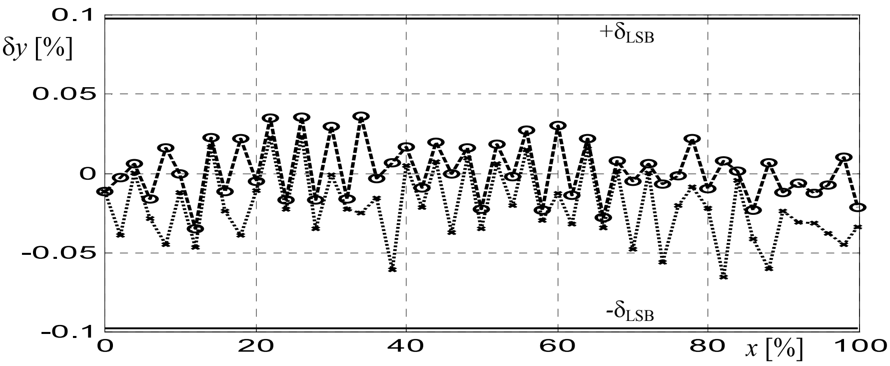

Fig. 3. Measurement error curve after correction using averaging. Maximum (dashed line with o marker) and minimum (dotted line with x marker) has been calculated after subtraction of I-element error.

In the next case, for every sample b in Eq. 2 average value from m samples has been calculated. Sampling the input x to obtain b_0 it is typical case of oversampling (with f_{s1} = 1201.9 Hz), where sufficient input noise is needed. The goal has also been to use faster RC-filter, which is now possible because of averaging of I-element output signal. This could be seen as mean value calculation of a periodic waveform, which determines the number of samples m=39 provided quasi-synchronous sampling. Fig. 3 shows that this implementation of iterative correction method is more accurate besides improved speed and expected tendency to I-element accuracy has been obtained due to reduced quantisation error. Total error after correction has been less than 0.5 LSB evaluated from average error values and 0.7 LSB from maximal errors after subtraction of I-element error.

4. Discussion

Parameters in Table 1 show iterative method functionality and also its enhancement through oversampling and averaging. Theoretically *ENOB* improvement should be 2.643 LSB with oversampling according to Eq. 4. This has not been achieved because of insufficient input noise (which explains fluctuating curve shape in the Fig. 3) and determining influence of I-element. The theory of oversampling listed above is not really satisfying in this case because AC part of I-element output is not white noise. The input noise can be increased by dithering which has been yet implemented and better measurement accuracy has been obtained.

Table 1. Correction parameters by simple sampling or averaging.

Parameter	*Mean SNR (dB)*	*Mean ENOB (LSB)*
Before correction	*49.1*	*8.2*
After correction with simple sampling	*56.2*	*9.3*
After correction with averaging	*59.7*	*9.9*
After correction with averaging and I-element error subtraction	*63.0*	*10.5*

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