Contribution of the SPRT Calibration to Uncertainty of Temperature T_{90} Measured by the Calibrated SPRT

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The propagation of uncertainties, when the International Temperature Scale of 1990 (ITS-90) is used by a standard platinum resistance thermometer (SPRT) calibrated at defining fixed points (DFP), can be solved by applying several approaches. The article presents an analysis of contribution of covariance between resistances of SPRT at the defining fixed points (DFP). Its effect on temperature measured by calibrated SPRT is demonstrated by using real calibration data.

Keywords: ITS-90, standard platinum resistance thermometer, defining fixed point, calibration, uncertainty

1. INTRODUCTION

THE DETERMINATION of the temperature T_{90} on the ITS-90 is based on the determination of the SPRT resistance ratio $W(T_{90}) = R(T_{90})/R_{\mathrm{TPW}}$, when $R(T_{90})$ is the resistance of the SPRT at the temperature T_{90} and R_{TPW} is its resistance at the temperature of triple point of water (273.16 K). The determination of SPRT uncertainties is based on the ISO *Guide to the Expression of Uncertainty in Measurement* (GUM) [2]. The various aspects of determination of uncertainties, when measuring temperature T_{90} by the calibrated SPRT, have been discussed in many publications. Solid models for both the identification and evaluation of typical uncertainty sources have been presented and they are routinely used by the SPRT users.

When determining $T_{90,}$ one of uncertainty sources, is the calibration of SPRT itself. The uncertainties of SPRT resistance at defining fixed points temperatures are propagated through interpolation equations of ITS-90 and contribute to the uncertainty of T_{90}. Moreover, the determination of R_{TPW} value for SPRT calibration/use also affects the result of measurement.

White [4], Mayer and Ripple [6] investigated several cases of R_{TPW} determination for SPRT calibration and use, but the covariance between resistances of the SPRT at the DFPs ($R_{\mathrm{DFP}i}$) was not included (except the covariance between R_{TPW}). In this paper we present models for calculating the SPRT calibration contribution to the uncertainty of temperature T_{90} measured by the calibrated SPRT ($u(T_{90})$) between fixed points, when covariance between $R_{\mathrm{DFP}i}$ is included.

We discuss the subrange of ITS-90 from 0 °C up to 660 °C. In this subrange, the SPRT is calibrated at the triple point of water, freezing point of tin, freezing point of zinc, and freezing point of aluminum [1].
The effect of covariance between $R_{\mathrm{DFP}i}$ is demonstrated by using the real calibration data.

2. CONTRIBUTION OF SPRT CALIBRATION TO U(T90)

Temperature T_{90} is defined by the SPRT reference function. For the range from 0 °C up to 961 °C it is:

$$T_{90}/K - 273.15 = D_0 + \sum_{i=1}^{9} D_i \left[\frac{W_{\mathrm{r}}(T_{90}) - 2.64}{1.64} \right]^i \qquad (1)$$

where the constants D_{i} are provided in [1] and values of $W_{\mathrm{r}}(T_{90})$ are determined from the deviation function.

$$\begin{aligned} W_{\mathrm{r}}(T_{90}) &= W(T_{90}) - \Delta W(T_{90}) = \\ & a(W(T_{90})-1) - b(W(T_{90})-1)^2 - c(W(T_{90})-1)^3 \end{aligned} \qquad (2)$$

$\Delta W(T_{90})$ is determined from the SPRT calibration.

The values of resistances $R_{\mathrm{DFP}i}$, their uncertainties $u(R_{\mathrm{DFP}i})$, and covariance between them $u(R_{\mathrm{DFP}i}, R_{\mathrm{DFP}j})$ are evaluated as a result of the SPRT calibration.
Temperature T_{90} is evaluated from the inverse function to the reference function (1).
Uncertainty of temperature T_{90} is evaluated by the following equation:

$$u(T_{90}) = A_{W\mathrm{r}T90}\, u(W_{\mathrm{r}}(T_{90})) \qquad (3)$$

where $A_{W\mathrm{r}T90}$ are sensitivity coefficients,
$u(W_{\mathrm{r}}(T_{90}))$ is a standard uncertainty of $W_{\mathrm{r}}(T_{90})$.

Sensitivity coefficient $A_{W\mathrm{r}T90}$ is evaluated by derivation of the function (1)

$$A_{\mathrm{r}T90} = \frac{\partial f(T_{90})}{\partial W_{\mathrm{r}}(T_{90})} = \sum_{i=1}^{9} \frac{i \cdot D_i}{1{,}64} \left[\frac{W_{\mathrm{r}}(T_{90}) - 2{,}64}{1{,}64} \right]^{i-1} \qquad (4)$$

Regarding the equation (2), $u(W_{\mathrm{r}}(T_{90}))$ is given by

$$u^2(W_r(T_{90})) = u^2(W(T_{90})) + u^2(\Delta W(T_{90})) - 2u(W(T_{90}), \Delta W(T_{90})) \quad (5)$$

Where $u(W_r(T_{90}))$ is a standard uncertainty of $W_r(T_{90})$, $u(\Delta W(T_{90}))$ is a standard uncertainty of $\Delta W(T_{90})$, $u(W(T_{90}), \Delta W(T_{90}))$ is covariance between $W(T_{90})$ and Δ $W(T_{90})$.

In the equations [7, 10] we presented a method for evaluating the SPRT calibration, when covariance between R_{DFPi} is included, and a method for evaluating the calibration contribution to uncertainty of T_{90} which is measured by the calibrated SPRT. The presented method was based on coefficients of the deviation function. In this article we present methods based on SPRT resistance ratios W_{DFPi} and resistances R_{DFPi}.

Regarding the equation (5), $u(W_r(T_{90}))$ is evaluated by the following
equation.

$$u^2(W_r(T_{90})) = u^2(W(T_{90})) + \sum_{i=2}^{4} A_{iT90}^2 u^2(W_{DFPi}) + 2\sum_{i=2}^{3}\sum_{i>j}^{4} A_{iT90} A_{jT90} u(W_{DFPi}, W_{DFPj}) + 2\sum_{i=2}^{4} A_{iT90} u(W(T_{90}), W_{DFPi}) \quad (6)$$

Where $$A_{iT90} = \frac{\partial W_r(T_{90})}{\partial W_{DPBi}}$$

Various methods, including algebraic approximations [6] would be used to calculate these derivations. Sensitivity coefficients would be evaluated also by using the Lagrange polynominal, as demonstrated at [3, 5].

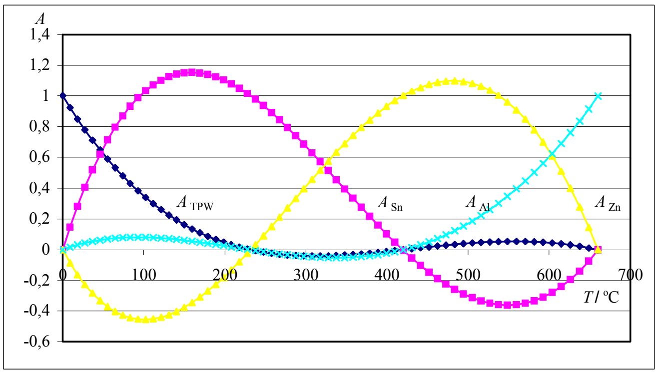

Fig.1. Sensitivity coefficients A_{TPW}, A_{Sn}, A_{Zn},, A_{Al}

Uncertainty of W_{DFPi} is given by

$$u^2(W_{DFPi}) = \frac{u^2(R_{DFPi}) + W_{DFPi}^2\, u^2(R_{TPWi})}{R^2{}_{TPWi}} \quad (7)$$

The covariance $u(W_{DFPi}, W_{DFPj})$ are as follows

$$u(W_{DFPi}, W_{DFPj}) = \frac{1}{R_{TPWi} R_{TPWj}}\left[u(R_{DFPi}, R_{DFPj}) - W_{DFPi} u(R_{TPWi}, R_{DFPj}) - W_{DFPj} u(R_{TPWj}, R_{DFPi}) + W_{DFPi} W_{DFPj} u(R_{TPWi}, R_{TPWj})\right] \quad (8)$$

$u(W(T_{90}), W_{DFPi})$ are given by

$$u(W(T_{90}), W_{DFPi}) = -\frac{1}{R_{TPW}} W_{T90} u(R_{TPW}, W_{DFPi}) \quad (9)$$

where $u(R_{TPW}, W_{DFPi})$

$$u(R_{TPW}, R_{DFPi}) = \frac{1}{3}\sum_{j=2}^{4} u(R_{TPW\,j}, R_{DFPi}) \quad (10)$$

A substitution of $u(W(T_{90}))$ from the equation (7), when index „i" is replaced by (T_{90}), $u(W_{DFPi})$ for i = 2,3,4 also from (7), $u(W_{DFPi}, W_{DFPj})$ from (8) and $u(W(T_{90}), W_{DFPi})$ from (9) (we assume that $u(R(T_{90})), R_{DFPi}) = u(R(T_{90})), R_{TPWi}) = 0$ and $R_{TPW1} \approx R_{TPW2} \approx R_{TPW3} \approx R_{TBV4}$) into the equation (5), results in

$$u^2(W_r(T_{90})) = \frac{1}{R_{TPW}^2}\Bigg[u^2(R(T_{90})) + W(T_{90})^2 u^2(R_{TPW}) + \sum_{i=2}^{4} A_{iT90}^2 \left[u^2(R_{DFPi}) + W_{DFPi}^2 u^2(R_{TPWi})\right] + 2\sum_{i=2}^{3}\sum_{j=i+1}^{4} A_{iT90} A_{jT90} u(R_{DFPi}, R_{DFPj}) + 2\sum_{i=2}^{3}\sum_{j=i+1}^{4} A_{iT90} A_{jT90} W_{DFPi} W_{DFPj} u(R_{TPWi}, R_{TPWj}) - 2\sum_{i=2}^{4}\sum_{j=2}^{4} A_{iT90} A_{jT90} W_{DFPj} u(R_{DFPi}, R_{TPWj}) - 2\sum_{i=2}^{4} A_{iT90} W(T_{90}) W_{DFPi} u(R_{TPW}, R_{TPWi}) + 2\sum_{i=2}^{4} A_{iT90} W(T_{90}) u(R_{TPW}, R_{DFPi})\Bigg] \quad (11)$$

The uncertainties and covariance in the equation (11) are, except for $u(R_{TPW}, R_{DFPi})$ and $u(R_{TPW}, R_{TPWi})$, determined in the SPRT calibration. The three different cases of $u(R_{TPW}, R_{DFPi})$ and $u(R_{TPW}, R_{TPWi})$ are described below.

a) R_{TPW} used for the SPRT calibration and R_{TPW} used for the determination of T_{90} (measurement) are determined from the independent no correlated measurements and also all the R_{DFPi} are determined from the independent no correlated measurements, i.e. $u(R_{TPW}, R_{DFPi}) = 0$, $u(R_{TPW}, R_{TWi}) = 0$, $u(R_{TPWi}, R_{TPWj}) = 0$, $u(R_{TPWi}, R_{DFPj}) = 0$, $u(R_{DFPi}, R_{DFPj}) = 0$. This is a rather theoretical case, because correlations between R_{DFPi} always exist.

$$u^2(W_r(T_{90})) = \frac{1}{R_{TPW}^2}\left[u^2(R(T_{90})) + W(T_{90})^2 u^2(R_{TPW}) + \sum_{i=2}^{4}\left(u^2(R_{DFPi}) + W_{DPBi}^2 u^2(R_{TPWi})\right) A_i^2\right] \quad (12)$$

b1) All the R_{TPWi} are from the single calibration of SPRT at TPW and R_{DFPi} are not correlated, i.e. $u(R_{TPW}, R_{TPWi}) = u^2(R_{TPW})$, $u(R_{TPW}, R_{DFPi}) = u^2(R_{TPW})$, $u(R_{TPW}, R_{DFPi}) = 0$, $u(R_{TPWi}, R_{DFPj}) = 0$, $u(R_{DFPi}, R_{DFPj}) = 0$.

$$u^2(W_r(T_{90})) = \frac{1}{R_{TPW}^2}\left[u^2(R(T_{90})) + \sum_{i=1}^{4} A_{iT90}^2 u^2(R_{DFPi})\right] \quad (13)$$

b2) All the $R_{\mathrm{TPW}i}$ are from the single calibration of SPRT at TPW and $R_{\mathrm{DFP}i}$ are correlated, i.e. $u(R_{\mathrm{TPW}}, R_{\mathrm{TPW}i}) = u^2(R_{\mathrm{TPW}})$, $u(R_{\mathrm{TPW}i}, \mathrm{R}_{\mathrm{TPW}j}) = u^2(R_{\mathrm{TPW}})$.

$$u^2(W_{\mathrm{r}}(T_{90})) = \frac{1}{R_{\mathrm{TPW}}^2}\Bigg[u^2(R(T_{90})) + W(T_{90})u^2(R_{\mathrm{TPW}}) + \sum_{i=2}^{4} A_{iT90}^2\left[u^2(R_{\mathrm{DFP}i}) + W_{\mathrm{DFP}i}^2 u^2(R_{\mathrm{TPW}i})\right] +$$
$$2\sum_{i=2}^{3}\sum_{j=i+1}^{4} A_{iT90}A_{jT90}u(R_{\mathrm{DFP}i}, R_{\mathrm{DFP}j}) + 2\sum_{i=2}^{3}\sum_{j=i+1}^{4} A_{iT90}A_{jT90}W_{\mathrm{DFP}i}W_{\mathrm{DFP}j}u^2(R_{\mathrm{TPW}}) -$$
$$2\sum_{i=2}^{4}\sum_{j=2}^{4} A_{iT90}A_{jT90}W_{\mathrm{DPB}j}u(R_{\mathrm{DFP}i}, R_{\mathrm{TPW}}) -$$
$$2\sum_{i=2}^{4} A_{iT90}W(T_{90})W_{\mathrm{DFP}i}u^2(R_{\mathrm{TPW}}) + 2\sum_{i=2}^{4} A_{iT90}W(T_{90})u(R_{\mathrm{TPW}}, R_{\mathrm{DFP}i})\Bigg] \tag{14}$$

c1) one of R_{TPW} is used for the calibration (denoted as $R_{\mathrm{TPW,cal}}$) and the other one for the measuring of T_{90}. $R_{\mathrm{DFP}i}$ are not correlated, i.e. $u(R_{\mathrm{TPW}}, R_{\mathrm{TPW}i}) = 0$, $u(R_{\mathrm{TPW}i}, R_{\mathrm{DFP}i}) = u^2(R_{\mathrm{TPW}})$, $u(R_{\mathrm{TPW}}, R_{\mathrm{DFP}i}) = 0$, $u(R_{\mathrm{TPW}i}, R_{\mathrm{DFP}j}) = 0$.

$$u^2(W_{\mathrm{r}}(T_{90})) = \frac{1}{R_{\mathrm{TPW}}^2}\cdot$$
$$\left[u^2(R(T_{90})) + W(T_{90})^2u^2(R_{\mathrm{TPW}}) + \sum_{i=2}^{4} A_{iT90}^2u^2(R_{\mathrm{DFP}i}) + (W(T_{90}) - A_{\mathrm{TPW}}(T_{90}))^2u^2(R_{\mathrm{TPW,cal}})\right] \tag{15}$$

c2) one of R_{TPW} is used for the calibration (denoted as $R_{\mathrm{TPW,cal}}$) and the other one for the measuring of T_{90}. $R_{\mathrm{DFP}i}$ are correlated,.i.e. $u(R_{\mathrm{TPW}}, R_{\mathrm{TPW}i}) = 0$, $u(R_{\mathrm{TPW}i}, R_{\mathrm{TPW}j}) = u^2(R_{\mathrm{TPW}})$, $u(R_{\mathrm{TPW}}, R_{\mathrm{DFP}i}) = 0$

$$u^2(W_{\mathrm{r}}(T_{90})) = \frac{1}{R_{\mathrm{TPW}}^2}\Big[u^2(R(T_{90})) + W^2(T_{90})u^2(R_{\mathrm{TPW}}) +$$
$$\sum_{i=2}^{4} A_{iT90}^2u^2(R_{\mathrm{DFP}i}) + (W(T_{90}) - A_{\mathrm{TPW}T})^2u^2(R_{\mathrm{TPWcal}}) +$$
$$2\sum_{i=2}^{3}\sum_{j=i+1}^{4} A_{iT90}A_{jT90}u(R_{\mathrm{DFP}i}, R_{\mathrm{DFP}j}) - 2\sum_{i=2}^{4}\sum_{j=2}^{4} A_{iT90}A_{jT90}W_ju(R_{\mathrm{DFP}i}, R_{\mathrm{TPW,cal}})\Big] \tag{16}$$

3. CALCULATIONS AND SUMMARY

The presented model was used with the real SPRT calibration data (calibration was performed at the Slovak Institute of Metrology). The calibration data are shown in the Tab.1, Tab.2, Tab.3, Tab.4, and Tab.5.
The considered uncertainty sources are:

- purity of the DFP substance /column 1 at Tab.3, Tab.4 and Tab.5/
- hydrostatic pressure /column 2/
- self-heating of SPRT /column 3/
- perturbing heat exchanges between the both sensor and surrounding parts different in temperature from the liquid-solid phase change /column 4/
- gas pressure in the cell /column 5/
- choice of fixed point value /column 6/
- isotopic composition (only for TPW) /column 7/
- residual gas pressure at the TPW cell /column 8/
- resistance of standard resistor /column 9/
- nonlinearity of resistance bridge /column 10/
- calibration of the standard resistance /column 11/

Table 1 SPRT calibration data.

DFB	$\boldsymbol{R_{\mathrm{DFP}}}$ **/ Ω**	$\boldsymbol{u(R_{\mathrm{DFP}})}$ **/ Ω**
$\mathrm{TPW_{Sn}}$	24.800200	$1.17\cdot10^{-5}$
$\mathrm{TPW_{Zn}}$	24.800193	$1.17\cdot10^{-5}$
$\mathrm{TPW_{Al}}$	24.800187	$1.17\cdot10^{-5}$
Sn	46.939753	$3.85\cdot10^{-5}$
Zn	63.705675	$4.98\cdot10^{-5}$
Al	83.719187	$6.32\cdot10^{-5}$

Table 2 Expanded uncertainties of the SPRT calibration at DFPs (k=2) / mK.

Source	1	2	3	4	5	6	7	8	9	10	11	B*	A*	C*
$u(R_{\mathrm{TPWSn}})$	-	4.00 $.10^{-7}$	2.00 $.10^{-6}$	2.00 $.10^{-6}$	-	-	4.90 $.10^{-6}$	1.50 $.10^{-9}$	2.0 $.10^{-7}$	2.00 $.10^{-6}$	9.89 $.10^{-6}$	1.258 $.10^{-5}$	1.98 $.10^{-6}$	1.273 $.10^{-5}$
$u(R_{\mathrm{TPWZn}})$	-	4,00 $.10^{-7}$	2,00 $.10^{-6}$	2,00 $.10^{-6}$	-	-	4.90 $.10^{-6}$	1.50 $.10^{-9}$	2.0 $.10^{-7}$	2.00 $.10^{-6}$	9.89 $.10^{-6}$	1.258 $.10^{-5}$	1.98 $.10^{-6}$	1.273 $.10^{-5}$
$u(R_{\mathrm{TBVAl}})$	-	4.00 $.10^{-7}$	2.00 $.10^{-6}$	2.00 $.10^{-6}$	-	-	4.90 $.10^{-6}$	1.50 $.10^{-9}$	2.0 $.10^{-7}$	2.00 $.10^{-6}$	9.89 $.10^{-6}$	1.258 $.10^{-5}$	1.98 $.10^{-6}$	1.273 $.10^{-5}$
$u(R_{\mathrm{Sn}})$	1.934 $.10^{-5}$	1.66 $.10^{-6}$	1.84 $.10^{-6}$	4.60 $.10^{-6}$	3.13 $.10^{-6}$	7.37 $.10^{-6}$	-	-	2.1 $.10^{-7}$	1.01 $.10^{-5}$	2.39 $.10^{-5}$	3.378 $.10^{-5}$	1.842 $.10^{-5}$	3.848 $.10^{-5}$
$u(R_{\mathrm{Zn}})$	3.12 $.10^{-5}$	1.91 $.10^{-6}$	1.73 $.10^{-6}$	4.33 $.10^{-6}$	3.81 $.10^{-6}$	6.93 $.10^{-6}$	-	-	2.7 $.10^{-7}$	9.53 $.10^{-6}$	3.21 $.10^{-5}$	4.67 $.10^{-5}$	1.734 $.10^{-5}$	4.981 $.10^{-5}$
$u(R_{\mathrm{Al}})$	3.974 $.10^{-5}$	1.03 $.10^{-6}$	1.59 $.10^{-6}$	7.95 $.10^{-6}$	5.64 $.10^{-6}$	7.95 $.10^{-6}$	-	-	3.6 $.10^{-7}$	1.03 $.10^{-5}$	3.97 $.10^{-5}$	5.854 $.10^{-5}$	2.385 $.10^{-5}$	6.321 $.10^{-5}$

A* - type A evaluation of standard uncertainty, B* - type B evaluation of uncertainty from the contributions 1-11, C* - combined standard uncertainty

Table 3. Correlation coefficients

Source	1	2	3	4	5	6	7	8	9	10	11
$r(R_{TPWSn},R_{TPWZn})$	1	1	1	1	-	-	1	1	1	1	1
$r(R_{TPWSn},R_{TPWAl})$	1	1	1	1	-	-	1	1	1	1	1
$r(R_{TPWZn},R_{TPWAl})$	1	1	1	1	-	-	1	1	1	1	1
$r(R_{TPWSn}, R_{Sn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWSn}, R_{Zn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWSn}, R_{Al})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWZn}, R_{Sn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWZn}, R_{Zn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWZn}, R_{Al})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWAl}, R_{Sn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWAl}, R_{Zn})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{TPWAl}, R_{Al})$	0	0	1	-1	0	0	-	-	1	1	1
$r(R_{Sn},R_{Zn})$	0	0	1	1	0	0	-	-	1	1	1
$r(R_{Sn}, R_{Al})$	0	0	1	1	0	0	-	-	1	1	1
$r(R_{Zn}, R_{Al})$	0	0	1	1	0	0	-	-	1	1	1

Table 4 Covariance on SPRT calibration at the DFPs

Source	1	2	3	4	5	6	7	8	9	10	11	Sum
$u(R_{TPWSn},R_{TPWZn})$	0	$1.6 .10^{-13}$	$4.0 .10^{-12}$	$4.0 .10^{-12}$	0	0	$2.40 .10^{-11}$	$2.25 .10^{-18}$	$4.0 .10^{-14}$	$4.0 .10^{-12}$	$9.787 .10^{-11}$	$1.341 .10^{-10}$
$u(R_{TPWSn},R_{TPWAl})$	0	$1.6 .10^{-13}$	$4.0 .10^{-12}$	$4.0 .10^{-12}$	0	0	$2.40 .10^{-11}$	$2.25 .10^{-18}$	$4.0 .10^{-14}$	$4.0 .10^{-12}$	$9.787 .10^{-11}$	$1.341 .10^{-10}$
$u(R_{TPWZn},R_{TPWAl})$	0	$1.6 .10^{-13}$	$4.0 .10^{-12}$	$4.0 .10^{-12}$	0	0	$2.40 .10^{-11}$	$2.25 .10^{-18}$	$4.0 .10^{-14}$	$4.0 .10^{-12}$	$9.787 .10^{-11}$	$1.341 .10^{-10}$
$u(R_{TPWSn}, R_{Sn})$	0	0	$3.7 .10^{-12}$	$-9.21 .10^{-12}$	0	0	0	0	$4.2 .10^{-14}$	$2.03 .10^{-11}$	$2.368 .10^{-10}$	$2.516 .10^{-10}$
$u(R_{TPWSn}, R_{Zn})$	0	0	$3.5 .10^{-12}$	$-8.67 .10^{-12}$	0	0	0	0	$5,4 .10^{-14}$	$1,91 .10^{-11}$	$3,173 .10^{-10}$	$3,312 .10^{-10}$
$u(R_{TPWSn}, R_{Al})$	0	0	$3.2 .10^{-12}$	$-1.59 .10^{-11}$	0	0	0	0	$7,2 .10^{-14}$	$2,07 .10^{-11}$	$3,931 .10^{-10}$	$4,012 .10^{-10}$
$u(R_{TPWZn}, R_{Sn})$	0	0	$3.7 .10^{-12}$	$-9.21 .10^{-12}$	0	0	0	0	$4,2 .10^{-14}$	$2,03 .10^{-11}$	$2,368 .10^{-10}$	$2,516 .10^{-10}$
$u(R_{TPWZn}, R_{Zn})$	0	0	$3.5 .10^{-12}$	$-8.67 .10^{-12}$	0	0	0	0	$5,4 .10^{-14}$	$1,91 .10^{-11}$	$3,173 .10^{-10}$	$3,312 .10^{-10}$
$u(R_{TPWZn}, R_{Al})$	0	0	$3.2 .10^{-12}$	$-1.59 .10^{-11}$	0	0	0	0	$7,2 .10^{-14}$	$2,07 .10^{-11}$	$3,931 .10^{-10}$	$4,012 .10^{-10}$
$u(R_{TPWAl}, R_{Sn})$	0	0	$3,7 .10^{-12}$	$-9,21 .10^{-12}$	0	0	0	0	$4,2 .10^{-14}$	$2,03 .10^{-11}$	$2,368 .10^{-10}$	$2,516 .10^{-10}$
$u(R_{TPWAl}, R_{Zn})$	0	0	$3,5 .10^{-12}$	$-8.67 .10^{-12}$	0	0	0	0	$5,4 .10^{-14}$	$1,91 .10^{-11}$	$3,173 .10^{-10}$	$3,312 .10^{-10}$
$u(R_{TPWAl}, R_{Al})$	0	0	$3,2 .10^{-12}$	$-1,59 .10^{-11}$	0	0	0	0	$7,2 .10^{-14}$	$2,07 .10^{-11}$	$3,931 .10^{-10}$	$4,012 .10^{-10}$
$u(R_{Sn},R_{Zn})$	0	0	$3,2 .10^{-12}$	$2,00 .10^{-11}$	0	0	0	0	$5,7 .10^{-14}$	$9,65 .10^{-11}$	$7,678 .10^{-10}$	$8,875 .10^{-10}$
$u(R_{Sn}, R_{Al})$	0	0	$2,9 .10^{-12}$	$3,66 .10^{-11}$	0	0	0	0	$7,56 .10^{-14}$	$1,05 .10^{-10}$	$9,514 .10^{-10}$	$1,096 .10^{-9}$
$u(R_{Zn}, R_{Al})$	0	0	$2,8 .10^{-12}$	$3,45 .10^{-11}$	0	0	0	0	$9,72 .10^{-14}$	$9,84 .10^{-11}$	$1,274 .10^{-9}$	$1,410 .10^{-9}$

Table 5 Correlation coefficients

$r(R_{TPWSn},R_{TPWZn})$	0.97
$r(R_{TPWSn},R_{TPWAl})$	0.97
$r(R_{TPWZn},R_{TPWAl})$	0.97
$r(R_{TPWSn},R_{Sn})$	0.56
$r(R_{TPWSn},R_{Zn})$	0.54
$r(R_{TPWSn},R_{Al})$	0.57
$r(R_{TPWZn},R_{Sn})$	0.56
$r(R_{TPWZn},R_{Zn})$	0.54
$r(R_{TPWZn},R_{Al})$	0.57
$r(R_{TPWAl},R_{Sn})$	0.56
$r(R_{TPWAl},R_{Zn})$	0.54
$r(R_{TPWAl},R_{Al})$	0.57
$r(R_{Sn},R_{Zn})$	0.46
$r(R_{Sn},R_{Al})$	0.45
$r(R_{Zn},R_{Al})$	0.45

We assess the effect of covariance from various origins to uncertainty of temperature T_{90}.
Individual contributions of terms of the equation (5) (or (11), (14)) to standard uncertainty, when $u(R(T_{90}))$ is not included, are shown in the Fig.2.
Fig.3 demonstrates the effect of covariance between R_{DFPi} to $u(T_{90})$. As we can see, it should not be neglected.

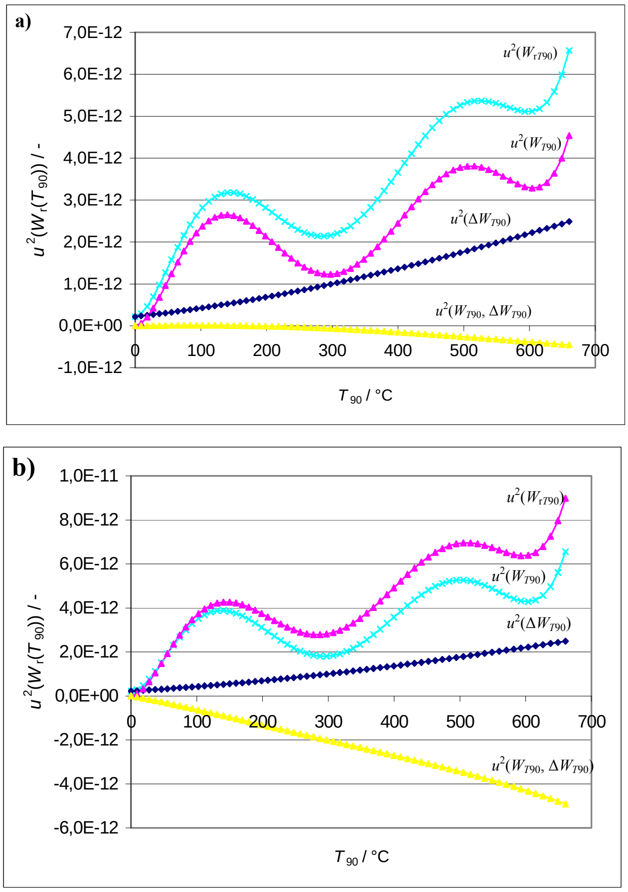

Fig. 2. Uncertainty of $W_r(T_{90})$ and its components, (see the equation (5)).

a) real calibration data (presented in tables above), when $r(R_{DFPi},R_{DFP})$=0.45 and $r(R_{TPWi},R_{TPWj})$=0.97, $r(R_{TPWi},R_{DFPj})$=0.55.

b) real calibration data (presented in tables above), when correlations between R_{DFPi} are not included, but correlations between R_{TPWi} are $r(R_{TBVi},R_{TBVj})$=0.97.

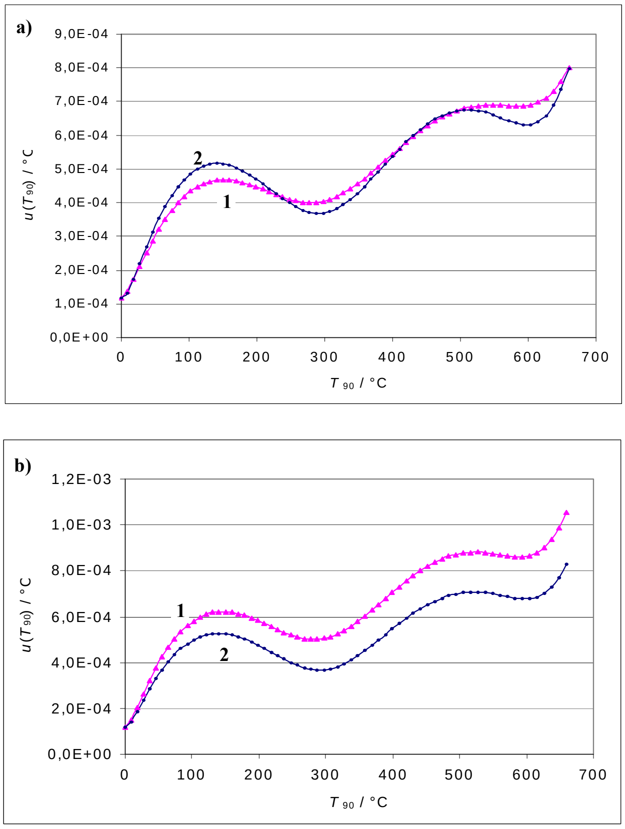

Fig.3. Calibration contribution to $u(T_{90})$, when covariance between R_{DFPi} is considered.

a) 1 – real calibration data, when $r(R_{DFPi},R_{DFPj})$=0.45 and $r(R_{TPWi},R_{TPWj})$=0,97, $r(R_{TPWi},R_{DFPj})$= 0.55, 2 – data from 1, when covariance between R_{DFPi} is not considered, but $r(R_{TPWi},R_{TPWj})$=0,97, $R_{TPW,cal}$ was used for the determination of T_{90} .

b) 1 - real calibration data when $r(R_{DFPi},R_{DFPj})$= 0.45, $r(R_{TPWi},R_{TPWj})$=0.97, $r(R_{TPWi},R_{DFPj})$= 0.55, 2 - data from 1, when correlations between R_{DFPi} are not considered, but $r(R_{TPWi},R_{TPWj})$=0.97, R_{TPW} was considered for the determination of T_{90}.

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