

# The Evaluation of Expanded Uncertainty of Measurement Results in Direct Measurements Using the LabVIEW Environment

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The paper presents a possible application of integrated LabVIEW environment to the final evaluation of measurement results in direct measurement. The possibilities of presenting and visualizing the uncertainty of measurement results in a convenient and user-friendly form are also discussed. The topics presented in the paper were using the selected LabVIEW application.

**Keywords:** Expanded uncertainty, coverage factor, confidence interval, probability distribution, virtual instrument

## 1. INTRODUCTION

WE ARE aware that, no measurement or test is perfect and the imperfections give rise to errors of measurement in the result. Consequently, the result of a measurement is only an approximation to the value of the measurand and is only complete when accompanied by a statement of the uncertainty of that approximation. An essentially appropriate concept was adopted, which is taken into consideration, that the method based on the command of the convolution of probability distributions of errors of components may be regarded as an exact method.

According to the document [1] the uncertainty of measurement result is a parameter, associated with the result of a measurement which characterizes the dispersion of the values that could reasonably be attributed to the measurand. The cause of the uncertainty of measurement result is the fact that we do not know the exact value of the measurand. This is because the results obtained in the process of direct measurement reveal errors in both the uncertainties of type A, which are the result of random effects, and the uncertainties of type B, caused by systematic effects.

There are many possible sources of uncertainty in testing, including:

- Incomplete definition of the test, the requirement is not clearly described,
- Imperfect realization of the test procedure,
- Inadequate knowledge of the effects of environmental conditions on the measurement process,
- Personal bias in reading analogue instruments,
- Changes in the characteristics or performance of a measuring instrument since the last calibration,
- Approximation and assumptions incorporated in the measurement method and procedure,
- Variations in repeated observations made under apparently identical conditions – such random effects may be caused by, for example: short-term fluctuations in local environment, e.g. temperature, humidity and air pressure, or variability in the performance of the tester.

During the measurement it may occur, however, that one of the uncertainties has a value small enough – as compared to the other one – that we can neglect it and consider only the

dominating uncertainty. It is assumed that we neglect a given uncertainty if its ratio to the other one is less than 10%.

It is obvious that expanded uncertainty  $U$  is a parameter allowing determining the limits of confidence interval comprising an unknown true value  $\mu$  with a defined probability  $\alpha$ :

$$P(\bar{x} - U < \mu < \bar{x} + U) = \alpha \quad (1)$$

where the expanded uncertainty  $U$ :

$$U = k(\alpha) \cdot u_c \quad (2)$$

- $u_c$  is the combined standard uncertainty, corresponding to the standard deviation of the distribution of errors considered,
- $k(\alpha)$  is the coverage factor, corresponding to the standardized variable of a given distribution.

To determine the confidence interval for an arbitrary parameter of population, it is necessary to know the probability distribution of the estimator of that parameter. In the considered case, it is necessary to determine the distribution of an average, which is the estimator of the unknown true value. The distribution of an average is the Student distribution, which for  $n$  tending to infinity becomes convergent to the normal distribution.

According to the recommendations of an international document [1] about the methods of evaluating and expressing uncertainty, the following notations and symbols will be adopted, corresponding to the parameters of probability distributions:

- standard uncertainty of type A -  $u_A$ , calculated on the basis of observed scatter of the results of a series of measurements, which is equal to the estimator of the standard deviation for average:

$$u_A = \sqrt{\frac{1}{n(n-1)} \sum_{i=1}^n (x_i - \bar{x})^2} = S_x \quad (3)$$

– standard uncertainty of type B -  $u_B$ , equal to the standard deviation of the assumed distribution of apparatus errors. With the assumption that the apparatus errors have rectangular distribution within the limits of maximum error  $\pm\Delta_g$ :

$$u_B = \frac{\Delta_g}{\sqrt{3}} = \sigma_J \quad (4)$$

– combined standard uncertainty for a directly measured value, when the standard uncertainties of type A and type B are taken into consideration:

$$u_C = \sqrt{u_A^2 + u_B^2} \quad (5)$$

## 2. LABVIEW PROGRAMMING ENVIRONMENT

LabVIEW (Laboratory Virtual Instrument Engineering Workbench) is graphical programming software used in developing programs for simulation, data acquisition, control, and communication application. In the LabVIEW environment, icons are interconnected to create a program generally referred to as a VI (Virtual Instrument) [2], [3], [4]. All VI's must have two components: the Front Panel and the Block Diagram. The Front Panel contains various controls and indicators while the Block Diagram includes a variety of functions. The functions (icons) are wired inside the Block Diagram, where the wires represent the flow of data. The execution of a VI is data dependent, which means that a node inside the Block Diagram will execute only if data is available at each input terminal of that node. By contrast, the execution of a traditional program, such as C program, follows the order in which the instructions are written [5]. VI's make up a new generation of measurement equipment. In these new devices the hardware realization of some functions is replaced with an appropriate program executed by a PC computer. It concerns mainly the device handling and the execution of signal processing algorithms. Thanks to those facilities, it is easy to modify such device and execute the signal processing algorithms. In particular, the LabVIEW environment seems to be commonly used at the university level teaching process for realization of specific programs for presentation of various problems.

### 3. VIRTUAL INSTRUMENT DESIGNED TO DETERMINE THE UNCERTAINTY OF MEASUREMENT RESULTS

A program was developed in the form of a virtual device, designed to determine the values of the uncertainty of measurement results. The prepared application makes it possible to determine the expanded uncertainty for direct measurement as well as to visualize the recorded measurement results [6].

After the application is run, a text file containing the measurement results should be read in. At this stage of working out the measurement results, we should assess whether some of the results do not reveal a gross error. If any of the results seems “doubtful”, we should delete it, giving the measurement number, and next run the program with the button “analysis”. Then the values of the limiting interval are

determined. If the “doubtful” result falls into the interval, we leave it, reversing its deletion, and we test the whole population. Otherwise such result should be rejected.

Fig.1 presents a fragment of the application panel, designed to verify gross errors. Another stage of the measurement is the parametric evaluation, i.e., determination of the values of the uncertainty of type A and type B, as well as the combined uncertainty, according to equations (3), (4) and (5).



Fig.1 Panel of virtual device designed to verify gross errors

At this stage, relations between the uncertainties of type A and type B should be defined. There are three options to choose from:  $u_A \gg u_B$ ,  $u_A \ll u_B$  or  $u_A \approx u_B$ . There are three bookmarks at our disposal in the program, which correspond to the above mentioned relations. Fig.2 presents an appropriate fragment of the panel.

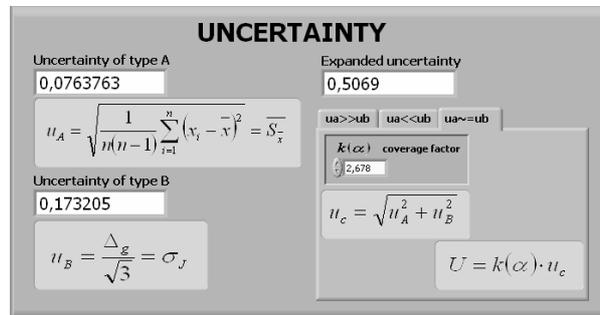


Fig.2 Panel of virtual device designed to evaluate relations between uncertainties errors

Fig.3 shows a fragment of the diagram of the application designed to determine the values of standard uncertainty of type A.

After the values of combined uncertainty are determined, the final stage of calculations is to determine the confidence interval, i.e., the value of expanded uncertainty  $U$ . The value of this uncertainty is related to coverage factor  $k(\alpha)$ .

When  $u_A \gg u_B$ , the value of probability  $\alpha$  should be given, and the application will automatically put in an appropriate value of the coverage factor for the Student distribution from a table contained in the program.

When  $u_B \gg u_A$ , the value of  $k(\alpha)$  is equal to the value of the coverage factor for the rectangular distribution. The application puts value of the factor in accordance with the value of probability  $\alpha$ .

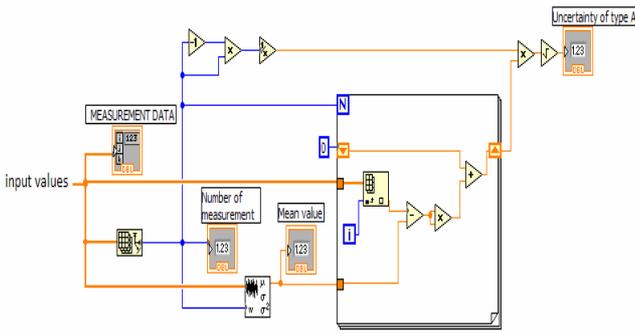


Fig.3 Fragment of application diagram designed to determine uncertainty of type A

For example, when  $u_A \approx u_B$ , we have a situation that the exact value of the coverage factor is equal to the factor for the convolution of the Student distribution and the rectangular distribution. In such case, we can either give the exact value of the factor – as long as we know it – or use one of the approximate evaluation methods of coverage factor, recommended in document [1].

This document suggests, for a measuring event with a small number of tests, a method according to which the coverage factor  $k(\alpha)$  assumes the values of standardized variable of Student distribution  $k_{m_e}$ , read from the table of this distribution for the method of effective number degrees of freedom  $m_e$ .

According to Welch-Satterthwaite Formula [7], if the combined standard uncertainty is a root of a sum of two or more variances estimated on the basis of results of a small number of tests with unknown standard deviation  $\sigma$ , the unknown distribution of the required standardized variable can be approximated by means of Student distribution for the effective number of degrees of freedom  $m_e$  given by the formula (6):

$$m_e = \frac{u_C^4}{\sum_{i=1}^n \left[ \left( \frac{\partial y}{\partial x_i} \right)^4 \frac{u^4(x_i)}{m_i} \right]} \quad (6)$$

Assuming that all partial derivatives are equal to one, in the considered case of direct measurement the effective number of degrees of freedom is described by means of relationship (7) resulting from the general Welch - Satterthwaite formula:

$$m_e = \frac{u_C^4}{\frac{1}{m_A} \cdot u_A^4 + \frac{1}{m_B} \cdot u_B^4} \quad (7)$$

where:

- $u_C$  is a standard combined uncertainty of value Y measured directly and computed according to the equation (5),
- $m_A = n - 1$  is the number of degrees of freedom,

- $m_B$  is the number of degrees of type B freedom and is computed on the basis of reliability of component standard uncertainty of type B.

In a situation when type B standard uncertainty is estimated on the basis of known rectangular distribution whose borders are defined by the limiting error of measuring devices, one can assume that this uncertainty is well known. Therefore, for the following analysis one can assume that the relative uncertainty of type B values equal to 0,1 [1], which reflects the number of degrees of freedom  $m_B = 50$ .

The same international document [1] recommends among other things to use arbitrarily imposed values of coverage factors which are equal appropriately  $k(\alpha) = 3$  for probability  $\alpha = 0.99$  and  $k(\alpha) = 2$  for probability  $\alpha = 0.95$ .

According to this assessment, factor  $k(\alpha)$  is attributed the value of the factor, which is close to the values of standardized variable of normal distribution.

Fig.4 shows the panel of a virtual device designed to evaluate the expanded uncertainty of measurement results in direct measurement in case when uncertainty of type A is comparable with uncertainty of type B -  $u_A \approx u_B$ . Coverage factor  $k(\alpha)$  was evaluated using the approximate method of effective number degrees of freedom.

#### 4. CONCLUSIONS

The paper presents the possibilities of utilizing the LabVIEW integrated environment in the process of evaluating the uncertainty of measurement results in direct measurement.

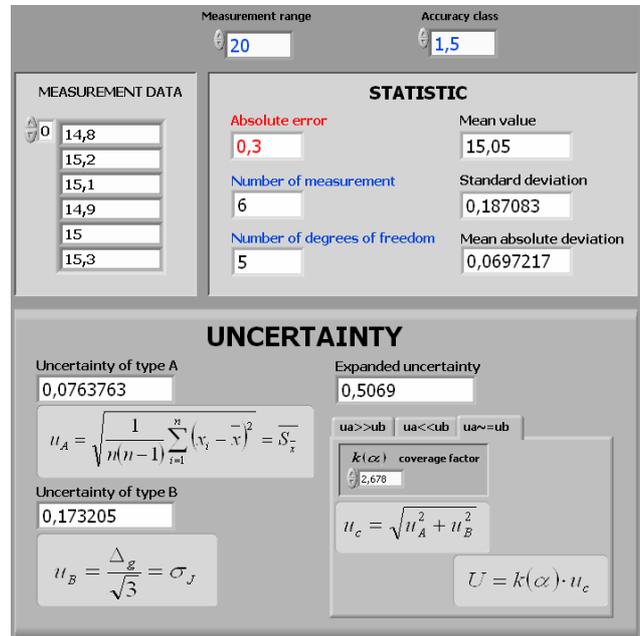


Fig.4 Panel of virtual device designed to determine expanded uncertainty of measurement result in direct measurement

The possibilities of presenting and visualizing the uncertainty of measurement results in a convenient and user-friendly form are discussed.

By using the LabVIEW environment – as it is illustrated with an example of the developed application – we can support or add variety to the teaching of students in the field of metrology and measurement theory.

The authors' present experience demonstrates that elaborate software is a very useful instrument assisting the teaching process. The application of computer simulation in teaching is a requirement of modern education.

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