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Precise length etalon controlled by stabilized frequency comb

R. Šmíd, O. Číp, J. Lazar

Department of Coherence Optics, Institute of Scientific Instruments of AS CR, v.v.i.,
Královopolská 147, 61264 Brno, Czech Republic, e-mail:smid@isibrno.cz

The progress in the field of optical frequency standards is oriented to femtosecond mode-locked lasers stabilized by technique of the optical frequency synthesis. Such a laser produces a number of coherent frequency components in certain interval of wavelengths. If we control the mode-locked laser by means of i.e. atomic clocks we ensure very stable frequency of these components. With respect to definition of SI unit "one meter" on the basis of speed of light, the stabilized mode-locked laser can be used for implementation of this definition in a non-traditional way. In the work we present our proposal of a system, which converts excellent frequency stability of components generated by the mode-locked laser to a net of discrete absolute lengths represented by a distance of two mirrors of an optical resonator.

Keywords: Fabry-Perot, interferometer, femtosecond, etalon

1. INTRODUCTION

METROLOGY in these days is based on precise optical frequencies. The precise frequency generator could be typically one of the He-Ne lasers stabilized in the order up to 10^{-13}.

Better frequency stability could be produced by frequency synthesis through pulsed femtosecond mode-locked lasers [1, 2]. Moreover this tool could span over a huge spectrum producing a frequency rule over the wide range of wavelengths. Using this clue one can find a new definition of length through ultrastable frequency comb spectra. The precise length standard is defined through Fabry-Perot interferometer (etalon) controlled by He-Ne stabilized laser [3]. We propose a novel method of defining length standard using a femtosecond frequency comb.

2. METHODOLOGY

A. Frequency Combs

Frequency combs are tools that consist of sources generating the femtosecond pulses. The train of pulses is characterized by the central wavelength, period of pulses, pulse shape and pulse to pulse phase shift. Femtosecond pulses are pulses typically in the order of 100 fs (up to 1 ps). Number of pulses in the train produces a frequency spectrum of comb lines around the central frequency (wavelength) [2]:

$$f_i = f_{ceo} + i\, f_{rep} \tag{1}$$

where i is the number of comb lines in the order of 10^6 and f_{rep}, and f_{ceo} are frequencies typically set in the radiofrequency domain, called repetition and offset frequency respectively. The repetition frequency f_{rep} is indirectly proportional to period of pulses and the offset frequency f_{ceo} describes phase shift. These two frequencies have naturally relative stability only up to 10^{-7} and form two dimensional structures which give us a chance to stabilize all frequency comb lines.

B. Methods of stabilization

Equation (1) illustrates the sensitivity of frequency comb lines to the stabilities of repetition frequency and offset frequency. The repetition frequency is very intensively presented in RF spectra and could be then easily detected.

The spectrum described by (1) is typically broad only up to 100 nm (10 to 20 THz). For stabilization it is convenient to broaden it by one optical octave where one can take the lower frequencies ("red") part and by second harmonic generation (SHG) produce doubled frequencies to "blue" part of the comb spectra. Respecting the (1) and applying simple math we can easily subtract the offset frequency:

$$2f_i - f_{2i} = f_{ceo} \tag{2}$$

Both f_{rep} and f_{ceo} could be stabilized by RF frequency standards, e.g. atomic clocks [4]. The atomic clocks could theoretically have relative frequency stability in RF frequency domain up to 10^{-15} which could be transferred to optical frequency domain.

The above mentioned method is the so called **f-to-2f** or **self-referencing method** of stabilization.

Another approach is based on stabilization of at least one comb component to the very stable continuous wave (cw) laser source e.g. iodine stabilized Nd:YAG [5] (with relative frequency stability up to 10^{-14}) or He-Ne lasers.

C. Fabry-Perot interferometer

Fabry-Perot interferometer or etalon (FPI) consist of two parallel mirrors. Two mirrors are plan-parallel in the simplest case.

FPI is an instrument which transfers precise frequencies characterized by the optical path distance between mirrors and by the mirror reflectance.

Such FPI is presented by a cavity with the length L_{cav} and has a periodic frequency transmission spectrum. Frequency of m-th longitudinal mode transmitted through the FPI can be expressed by:

$$f_m = \frac{mc}{2nL_{cav}} \tag{3}$$

where c is speed of light, n is index of refraction of medium in the cavity, and $2L_{cav}$ is one round trip of the FPI.

Index of refraction and following dispersion bring us new problems in the further analyses. Let us consider vacuum FPI for simplicity. Intermode distance of Fabry-Perot cavity is related through (3) to the distance between mirrors L_x.

$$f_{cav} = \frac{c}{2L_x} \tag{4}$$

Each transmitted light then fulfills the condition

$$f_{FP} = i\, f_{cav} \tag{5}$$

Periodical changes in distance between mirrors characterized by L_x to $L_x+\Delta L_x$ (using e.g. the piezo-electric transducer) with detector on the output mirror produce spectra of the so called scanning Fabry-Perot interferometer which enable us to measure broader spectra.

D. Transmission of comb lines through FPI

It has been shown that only a discrete value of frequency could be transmitted through FPI. If there is a relation that fulfills the condition

$$i\, f_{rep} = j\, f_{cav} \tag{6}$$

or in other words

$$f_{cav} = x\, f_{rep} \tag{7}$$

where *x*, *i*, and *j* are the integers. Each optical frequency comb line could be transferred through the FPI but the optical path distance between the mirrors should strictly respect the following equation:

$$L \; = \frac{c}{2xf_{rep}} \tag{8}$$

where L_x is optical path distance in the FPI. In frequency domain it represents laser spectral comb lines coinciding with transmission of the FPI. Fig. 1 represents a structure of such an optical path distances which corresponds to an optical frequency comb with f_{rep}=100 MHz.

E. Time domain analysis

The structure of the problem in the time domain is more complicated since in typical FPI we suppose continuous wave sources targeting the mirrors. In our case the source has in the simplest a structure of "continuous wave source" central wavelength modified by the pulse repetition and pulse shaping.

Strictly speaking the pulses have no central wavelength but a number of wavelengths since it changes its direct position during time, source errors, and since each curve in time could be described in frequency domain as a sum of numbers of frequencies according to Fourier transform theory.

Pulse in the time domain could be explained by [3].

$$E_p(t) = E_{env}(t)\cdot e^{i(\omega_c t+\phi_{ce})} \tag{9}$$

where E_{env} is envelope function of the pulse and ω_c and ϕ_{ce} are the carrier angular frequency and carrier-envelope phase, respectively. ϕ_{ce} for *i*-th pulse is given by $\phi_{ce} = i\,\Delta\phi_{ce} + \phi_0$

The train of *y* pulses travelling from the frequency comb source is described by the following relation

$$E(t) = \sum_y E_p(t - y\,\tau) \tag{10}$$

where $\tau = 1/f_{rep}$.

The relation between time and frequency domain quantities is covered by

$$f_{ceo} = -\frac{\Delta\phi_{ce}\, f_{rep}}{2\pi} \tag{11}$$

Pulses travel through Fabry-Perot and *i*-th "meets" *(i+1)* -th pulse after *j* round-trips.

This simple approach is valid only in a totally evacuated medium. Generally, refraction index is then frequency dependent according to this equation:

$$n = c\left(\frac{1}{v_g} + \frac{k_0}{\omega}\right) \tag{12}$$

where v_g is group velocity and k_0 is wavevector. Thus the relation (11) could not be completely fulfilled.

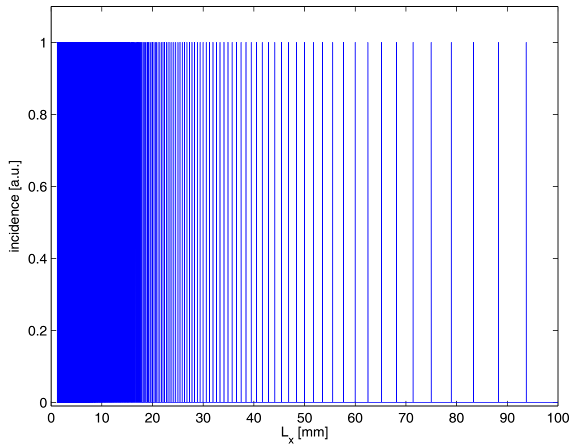

Fig.1 Places of incidence between spectral components of the optical resonator and mode-locked laser vs. resonator length L_x. The calculation is done for the mode-locked laser with the repetition frequency f_{rep} =100 MHz. The refractive index in the cavity is not considered.

The effect of dispersion and carrier-phase should be counted in resulting interference of pulses inside the cavity. If one pulse overlaps through the next in the FPI they interfere producing a structure of maxima and minima – the interference fringes which envelope corresponds to the pulse shape and pulse length. The higher the number of incidence pulses and the quality of the Fabry-Perot cavity, the stronger the contrast around the well defined precise length (according to the Fig.1). The precise length is at the extreme of the envelope (the fringe with maximal or minimal intensity).

3. EXPERIMENT AND RESULTS

In our first experiment, done for verification of our method, we used fiber based femtosecond laser TC1500 MenloSystems, GmbH. This laser has pulse length <100 fs, central wavelength 1560 nm and the repetition frequency 100 MHz (tunable +129 kHz, –257 kHz).

For this reason we have made FPI with two plan-parallel mirrors with intracavity length close to L_x = 375 mm with inter-mode FPI frequency 400 MHz which fulfills the condition (6), (7) and (8).

We propose a comparison method to find precise length L_x and follow the inter-mode frequency f_{cav}. We applied two tunable single-mode He-Ne lasers working at different wavelengths as the mode-locked laser works. Each of these lasers is locked to one of the neighboring longitudinal modes of the resonator. The frequency of beats between these two lasers determines the inter-mode frequency f_{cav}.

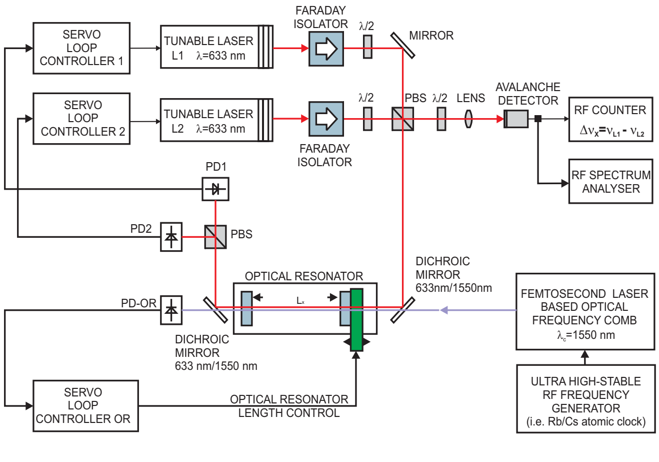

Fig.2 The schematic diagram. λ/2 are half-wave plates for 633 nm, L1 and L2 are He-Ne lasers used for measurement of inter-mode frequency f_{cav}, and OR is abreviation of the optical resonator.

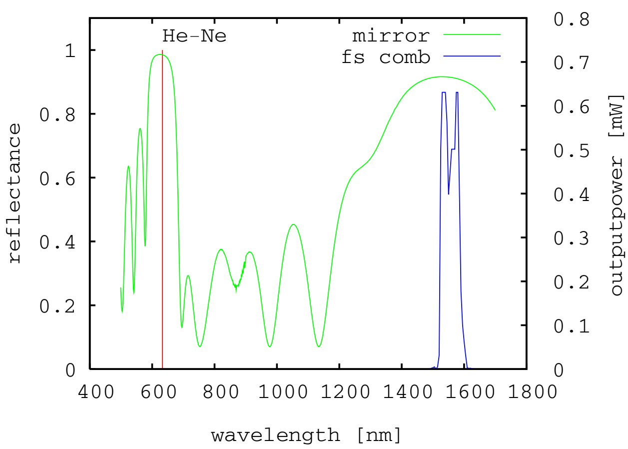

Fig.3 The reflectance of dichroic mirrors and output lines of He-Ne lasers and the frequency comb.

The schematic diagram of the whole optical and electronic arrangement is in Fig.2. We detected two laser beams produced by He-Ne lasers L1 and L2 into the same axis of propagation on the detector PD1 and PD2. L1 and L2 lasers are separated optically by different polarization. We used polarizing beam splitters PBS and set of half-wave plates for this purpose. FPI consist of two dichroic mirrors with different reflectance: 0.99 at 633 nm and 0.90 at 1560 nm (see Fig.3).

To lock the optical frequency of each laser L1 and L2 to neighboring longitudinal modes of the optical resonator we used a harmonics detection technique known at the laser spectroscopy [6,7].

A. First recorded data

After the locking of both lasers L1 and L2 to neighboring longitudinal modes we tuned the length of the cavity L_{cav} precisely with respect to possible range of repetition frequency of the mode-locked laser f_{rep}=100.000 MHz (+129 kHz, –257 kHz). We reached the inter-mode frequency f_{cav} = 399.7500 MHz.

The recordings of the spectral profile of the resonator on the photodetector PD1 and PD2 are in Fig.4. Its 1st harmonics are presented in Fig.5 (one harmonic is presented in opposite sign for easier reading).

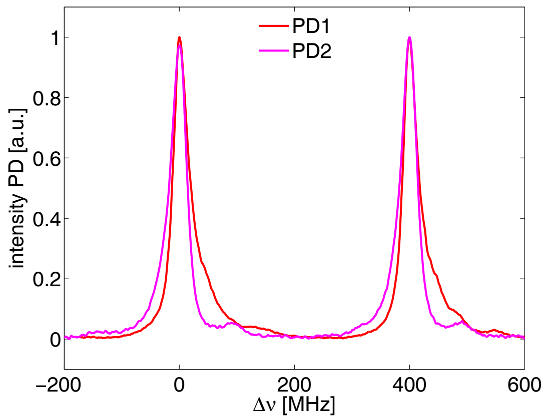

Fig.4 The spectral profiles of the optical resonator measured by scanning of the laser L1 (PD1) and L2 (PD2) along two neighboring longitudinal modes of the optical resonator with respect to one mode.

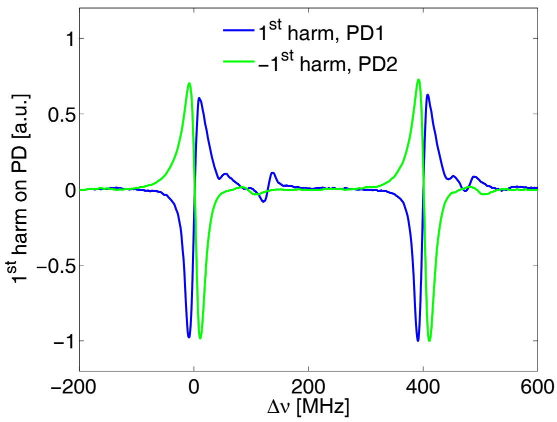

Fig.5 1st harmonics of the spectral profiles of the optical resonator measured by scanning of the laser L1 and L2 along two neighboring longitudinal modes of the optical resonator.

On the basis of the eq. (6), (7), and (8) we tuned the repetition frequency of the mode-locked laser to f_{rep} = 99.9375 MHz with respect to selected x = 4. When we approached this theoretically calculated value we observed interference for 1560 nm femtosecond pulsed beam in the output of the optical resonator. The profile of the interference is shown in Fig.6.

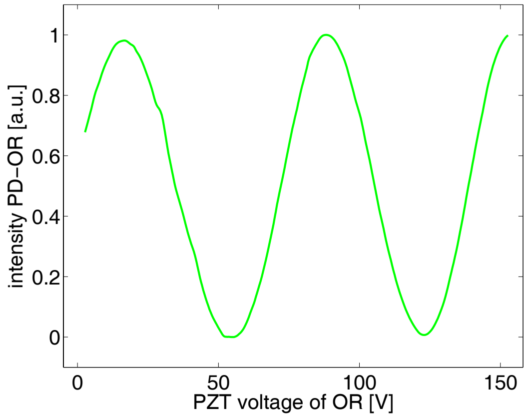

Fig.6 The record of the interference of femtosecond laser on the output of the optical resonator.

This was recorded during Fabry-Perot scanning mode. Picture shows the interference fringes with the pulses that are generated by cross-correlation of *i*-th pulse on the *(i+1)* -th.

Then we tuned the repetition frequency of the mode-locked laser to the extreme of the interference. The frequency f_{rep} = 99.9398 MHz was readout for this extreme.

After calculation of the true value of x we obtain $k=f_{cav}/f_{rep}$=3.99991. The deviation from theoretically expected value x=4 is caused probably by a different refractive index of air for 633 nm and 1560 nm inside the cavity. According to [8] the refractive index of air is:

n (632.8 nm, t=26°C, 101.325 kPa, 60 % humidity)=1.000265614±0.000000033,

n ((1560±20) nm, t=26°C, 101.325 kPa, 60% humidity)=1.00026244±0.00000005.

The difference caused by the different refraction index of air at 633 nm and 1560 nm is $\Delta n(\Delta L) = (3.17 \pm 0.06) \cdot 10^{-6}$, which gives us a relative inaccuracy of the order 10^{-6}. This inaccuracy is one order lower compared to measured value. Other sources of noise to the measured value could be the cavity configuration, beam radius and the beam focal length, mirror reflectance.

The effect of refractive index of air could be possibly removed by evacuation and isolation of Fabry-Perot cavity.

4. Conclusion

Our experiments proved the possibility of direct transfer between highly precise and stable radio frequency etalons (ev. optical frequency etalons) into discrete lengths. We adjusted the cavity length to be proportional to the repetition frequency of the mode-locked laser using two lasers locked to two neighboring longitudinal modes of the cavity. This allowed us to measure, control and adjust precisely the cavity mode spacing by monitoring of the beat frequency between the two lasers. The cavity, following resonant frequencies of the pulsed femtosecond laser, allowed positioning over precise discrete distances. Nevertheless further improvements are necessary, as Fabry-Perot cavity improvements and its isolation from ambient air.

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