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Information on Theoretical Investigations in Measurement, Particularly on Reading of Instruments

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The paper presents the results of investigations with a group of students dealing with the accuracy of reading of instruments, using information theory. After a short introduction to information theory and Shannon´s channel capacity, the relation between the reading error and the reading time is discussed, leading to the results with reference to the limiting (critical) frequencies obtainable. Using these results, the channel capacity may be calculated. When these quantities of the channel capacity are compared with other values obtained from the literature it follows the same order of magnitude of 10 bits per second. Finally, using information theory again, the consequences are deduced. Due to the development of digital methods in microelectronics with decreasing approximation errors as well as with decreasing cut-off and aliasing errors will be the future. This is especially important for direct-coupled digital sensors, which have been used increasingly.

Keywords: reading of instruments, information theory, channel capacity

1. Introduction

ABOUT half a century ago Shannon founded his information theory [1]. Originally developed for the application in communication, now it is used in various fields of science, thus also in measurement and instrumentation [2, 3, 4].

The channel capacity in particular combines two aspects of quality - describing both the static or statistical and the dynamic behaviour – into one more general quality criterion. Using this method, it is possible to compare the efficiency of different procedures of information acquisition including human perception. The paper addresses an important example in the field of measurement, namely the problem related to reading of instruments. Further, the consequences for the development of measurement, using information theory again, are outlined.

2. Channel Capacity as Quality Criterion

Fifty years ago, Shannon introduced the channel capacity C_t as a measure of the maximum possible information flow - i.e. under the condition of optimal coding [1]

$$C_t = f_c \text{ lb } (P_x / P_z + 1) \qquad (1a)$$

with the critical frequency f_c or due to the sampling theorem [5] the response time $t_r = 1/2f_c$ as a measure of the dynamic behaviour and the signal-to-noise ratio P_x / P_z as a measure of the static or statistical behaviour.

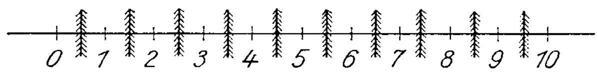

Fig.1 Explanation of the number of distinguishable amplitude steps for $m_a = 10$

An intuitive approach explains the number under the binary logarithm to be the number of distinguishable power steps m_p and consequently the square root of the number of amplitude steps m_a as shown in Figure 1 [2;3;4;5;6]. This means that the number of bits per second follows with the number of values per second $z = 1/t_r = 2f_c$ due to the sampling theorem

$$C_t = z \text{ lb } m_a = 2f_c \text{ lb } \sqrt{(P_x / P_z + 1)} = f_c \text{ lb } (P_x / P_z + 1) \qquad (1b)$$

Another more general approach uses a quality criterion Q

$$Q = \lambda (P_x / P_z ; f_c) = \lambda_1 (P_x / P_z) + \lambda_2 (f_c) \qquad (2a)$$

With

$\lambda_1 = \log f_c$
$\lambda_2 = \log [\text{ lb } (P_x / P_z + 1)]$

one obtains
$Q = \log f_c + \log [\text{ lb } (P_x / P_z + 1)] = \log [f_c \text{ lb } (P_x / P_z + 1)]$
and especially for

$$C_t = 10^Q = f_c \text{ lb } (P_x / P_z + 1) \qquad (2b)$$

Shannon´s channel capacity.

Some ten years ago it was hoped that by means of this method it would be possible to include also semantic aspects into a more general information theory, but this hope has not been fulfilled.

3. READING OF INSTRUMENTS

A group of students read the instruments with constant as well as with time variant values. The time during which the value was visible varied due to varying exposure time and thus the error of reading F_r as a function of the reading time t_r was found out. The results obtained while using, for instance, a moving-coil instrument, are given in Figure 2.

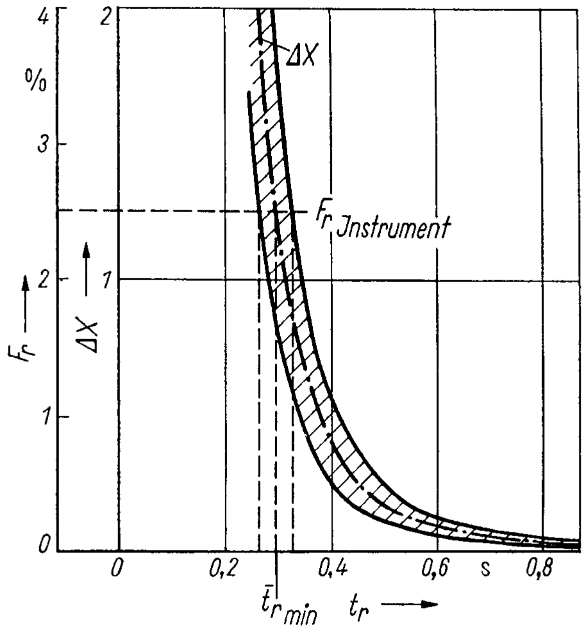

Fig. 2 Reading error F_r as a function of the reading time t_r for a moving-coil instrument

Due to the sampling theorem, these values - error and reading time – allow, to obtain the relation between the critical frequency f_c and the error F_r as shown in Figure 3.

Using these results, it is possible to calculate the channel capacity C_t as given in Equation (1b) with $z = 1/ t_r$ and $m_a = 1/ F_r$.

It was found that the channel capacity with good conditions, including optimal scale, illumination and distance between the students and the instrument, is about 15 bit/s. The value is similar to that found in the literature on the channel capacity of the human information acquisition [5,7].

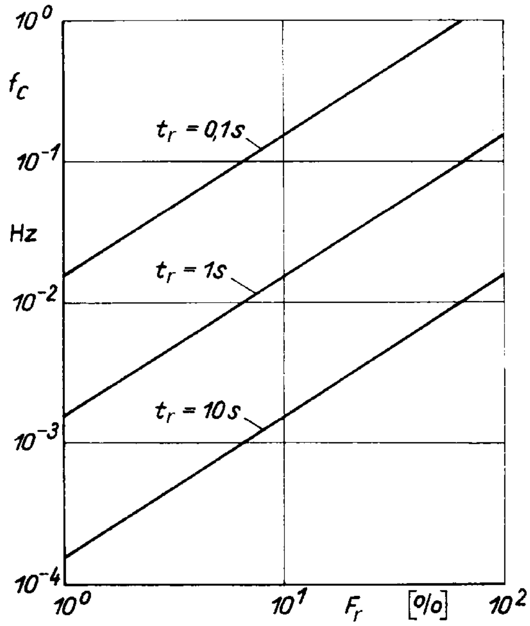

Fig. 3 Critical frequency f_c as a function of the error F_r for the case of reading of instruments; t_r = reading time

4. CONSEQUENCES FOR THE DEVELOPMENT OF MEASUREMENT

Looking to other values of the channel capacity of the total information chain of measuring systems, including signal processing devices, one obtains 100 kbit/s ....10 Mbit/s [5]. If these values are compared, it is clear that the human perception is the very smallest point in the whole measurement system. Therefore, automatic data acquisition instead of reading of instruments has been used increasingly. Above all, digital methods are applied today. Especially sensors with digital output are important because they can be coupled directly with the signal processing. In these cases, the low-pass-filtering necessary to fulfil the sampling theorem is often not possible because of the direct coupling of the sensor to the process. So aliasing errors are often unavoidable [8:9]. Furthermore, it should be accentuated that these aliasing errors depend on the processing after sampling e.g. on the algorithm of the digital processing [9].

To gain an impression of the further development the limitations between analogue and digital methods may be investigated. To compare both methods, once again the channel capacity is used as a suitable quality criterion.

In analogue methods, the number of distinguishable amplitude steps m is - if F is the relative amplitude error [2,5]

$$m = 1 + 1/ (2F) \tag{3a}$$

and thus the channel capacity follows

$$C_{t\,an} = 1 / t_r \; \mathrm{lb}\; m = 2f_c \;\mathrm{lb}\; (1 + 1 / 2F) \qquad (3b)$$

In the case of digital methods, pulses with the pulse frequency f_i are counted during the time T. This means that the number of distinguishable steps m is

$$m = 1 + Tf_i = 1 + f_i / (2f_c) \qquad (3c)$$

and the channel capacity is

$$C_{t\;dig} = 1/T \;\mathrm{lb}\; m = 2f_c \;\mathrm{lb}\; (1 + f_i / (2f_c) \qquad (3d)$$

Both values are equal for

$$Tf_i = f_i / 2f_c = 1 / 2F = m – 1 \qquad (4)$$

Figure 1 shows the results.

We will now take into consideration the tendency in the development of microelectronics. As is well-known from the last three decades, every two to three years a new generation of technology is used, leading to a reduction of the dimensions by the factor $1/\sqrt{2}$. This leads to the fact of an increase in the degree of integration by the factor 2 and a decrease in the areas by ½. This means that the capacities C are also reduced to half the values. As the time constants are T=RC, possible pulse frequencies are doubled from generation to generation [10]. Consequently, every 7 years the processing speed is increased by the factor 10.

On the other hand, due to the accession of the pulse frequency it follows that higher critical frequencies of the input signal are allowed e.g. that aliasing errors are of decreasing influence. The same tendency concerns the cut-off errors due to the higher sampling rate. Oversampling is also possible.

The same general assessment concerns the errors due to the approximation of the algorithms used for processing: Because of the on-line condition, the running time of an algorithm has to be shorter than the sampling period. So it follows that the higher the pulse rate and the higher the processing speed, the smaller the time necessary for operation of the algorithm. On the other hand, the more time is available, the smaller the approximation error of an algorithm. This leads to the conclusion that due to the development of microelectronics, the importance of approximation errors in the future will decrease because better algorithms can be realized.

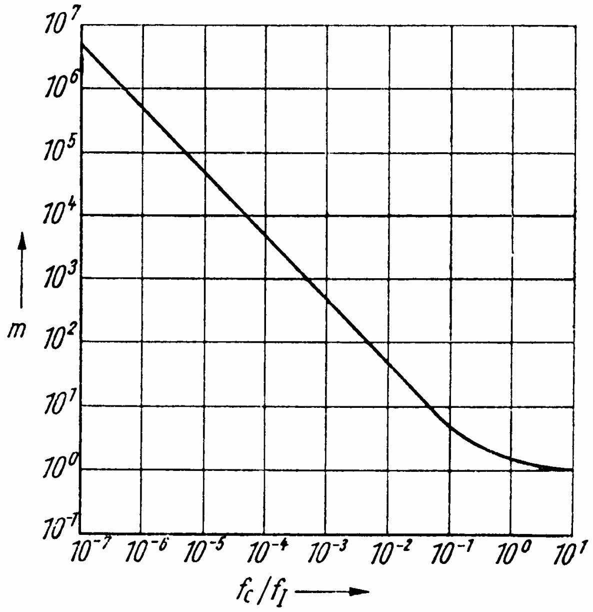

Fig. 4 Values of equal channel capacities of analogue and digital methods [5].
m = number of amplitude steps; f_c / f_I = relation of critical frequency to pulse frequency

5. CONCLUSION

First, it is shown how two different parts of errors - the static or statistical and the dynamic part – as a measure of the quality - may be combined into one quality criterion, *Shannon´s* channel capacity. Also, an intuitive approach using the number of distinguishable amplitude steps and the number of values per second is given.

The application of this method to the problem of reading of instruments, using experiments with a group of students, leads to the result that the value of the channel capacity lies in the order of magnitude of 10. A comparison of this result with the values given in the literature shows conformity with other fields of information acquisition by the humans. So, the investigations confirm the fact that the man-machine communication is the most critical point of information acquisition.

This fact is one of the reasons why automatic data acquisition has been used ever more frequently. To show the limitations of both the analogue and digital methods, the results of information theory once again were applied. With respect to the development of microelectronics, these investigations suggest that aliasing and cut-off errors as approximation errors will also play a decreasing role in the future, because of the increasing pulse frequencies and the decreasing time necessary to run the signal processing algorithms.

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