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The Magnetoplasmic Measurements of the Carrier Density in Many-Component Solid State Plasma

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Many semiconductor materials manufactured by help of nanotechnology have charge carriers of different type and mobility. Already existing carrier density and mobility measurement methods are not accurate enough for the case of several carrier components. The use of magnetoplasmic waves provides a simple and the most precise way to determine the density and mobility of each type of the carriers (electrons and/or holes).
Magnetoplasmic waves may be excited in semiconductors when the strong magnetic field H is applied. The semiconductor sample becomes partially transparent under these conditions. In the case of magnetoplasmic resonance within each of the carrier groups, the transparency coefficient has a maximum. For fixed values H and excitation frequency ω the density and mobility of every carrier type can be found.
Dispersion relation for two types of charge carriers is obtained and resonance curves are calculated.

Keywords: Magnetic measurement, imaging, magnetic susceptibility, helicons, calculation, microwave frequencies

1. INTRODUCTION

MANY SEMICONDUCTOR materials manufactured by help of nanotechnology have the charge of different type and mobility. Already existing carrier density and mobility measurement methods are not accurate enough in the case of several carrier components. The use of magnetoplasmic waves (helicons) provides a simple and more precise way to determine the density and mobility of each type of the carriers (electrons and/or holes).

Magnetoplasmic waves may be excited in semiconductors when the strong magnetic field H is applied and large Hall currents may exist. In the case of magnetoplasmic resonance within each of the carrier groups, the transparency coefficient has a maximum. For fixed values H and excitation frequency ω the density and mobility of every carrier type can be found.

2. SUBJECT & METHODS

Let us consider a semiconducting plate with an electric coil placed on its surface. For the sake of simplicity, we shall assume the semiconductor to be infinitely large. Let the semiconductor surface be parallel to the xy plane and the z-axis be directed perpendicular to it. Then the electrical current has only one component along the y-axis, and the magnetic has one along the x-axis. We assume, that the field H_x varies with time according to a harmonic law

$$\mathrm{H}_x = H\cos\omega t, |z| = a \tag{1}$$

Where a is the thickness of the plate along z-axis [1].

The currents induced in the semiconductor sample are directed in such a way so as to counteract the penetration of the field. As a result the varying magnetic field within the semiconductor will be other than zero only to a certain depth (skin depth).

If the semiconductor plate is at the same time placed into the strong magnetic field $H_x = H_0$, the Hall currents j_x appear and helicon magnetoplasmic waves may be excited.

Assuming that the conductivity of the plate is provided by electrons with an isotropic mass we have the following equations of motion for the current components j_x and j_y

$$\begin{aligned} \frac{d}{dt} j_x + \tau^{-1} j_x - \frac{eH_0}{mc} j_y = \frac{Ne^2}{m} E_x \\ \frac{d}{dt} j_y + \tau^{-1} j_y + \frac{eH_0}{mc} j_x = \frac{Ne^2}{m} E_y \end{aligned} \tag{2}$$

where N, e, m and τ are the density, charge, mass and collision time of the electrons, E_x and E_y, are the components of varying electrical field, and H_0 is the static magnetic field along the z-axis.

The equations of motion (2) must be solved along with the Maxwell equations

$$\begin{aligned} rot\vec{\varepsilon} = -\frac{1}{c}\frac{\partial \vec{\mathrm{H}}}{\partial t} \\ rot\vec{\mathrm{H}} = -\frac{1}{c}\frac{\partial \vec{\varepsilon}}{\partial t} + \frac{4\pi}{c}\vec{j} \end{aligned} \tag{3}$$

The solutions of the system of equations (2), (3) were sought in the form [2]

$$\vec{\varepsilon} = \mathrm{Re}\,\vec{E}\exp(-i\omega t),\ \mathrm{H} = \mathrm{Re}\,\vec{H}\exp(-i\omega t),$$
$$\vec{j} = \mathrm{Re}\,\vec{I}\exp(-i\omega t), \tag{4}$$

where the complex amplitudes $\vec{E},\vec{H}$ and $\vec{I}$ depend only on coordinate z and have the components x and y.

Taking into account that $\omega << \tau^{-1}$ = 10^{11} – 10^{13} Hz and ignoring the displacement currents in comparison with conductive ones we obtain a system of equations for the complex amplitudes of the varying magnetic field H_x and H_y

$$c^2\frac{\partial^2}{\partial z^2}H_x + i\omega\delta_{11}H_x + i\omega\delta_{12}H_y = 0$$
$$c^2\frac{\partial^2}{\partial z^2}H_y + i\omega\delta_{22}H_y - i\omega\delta_{12}H_x = 0 \tag{5}$$
,

where the components of the conductivity tensor σ are

$$\sigma_{11} = \sigma_{22} = \sigma_0\frac{1}{1+\left(\frac{eH_0\tau}{mc}\right)^2},$$
$$\sigma_{12} = \delta_0\frac{\frac{eH_0\tau}{mc}}{1+\left(\frac{eH_0\tau}{mc}\right)^2},\ \sigma_0 = \frac{Ne^2\tau}{m}. \tag{6}$$

The boundary conditions have the form

$$H_x = H, H_y = 0 \text{ if } z = \pm a,$$
$$\frac{\partial H_x}{\partial z} = \frac{\partial H_y}{\partial z} = 0 \text{ if } z = 0 \tag{7}$$
,

and the solutions of (5) satisfying (7) are

$$H_x = \frac{1}{2}H\left(\frac{\cos k_- z}{\cos k_- a} + \frac{\cos k_+ z}{\cos k_+ a}\right),$$
$$H_y = \frac{1}{2}iH\left(\frac{\cos k_+ z}{\cos k_+ a} + \frac{\cos k_- z}{\cos k_- a}\right), \tag{8}$$

where $k\pm$ can be detected from the characteristic equation

$$c^2k_\pm^{\ 2} = \frac{\omega_p^{\ 2}\omega}{\pm\omega_H + i\tau^{-1}}, \omega_p^{\ 2} = \frac{4\pi Ne^2}{m}, \omega_H = \frac{eH_0}{mc}. \tag{9}$$

The items with the argument k_- are caused by the helicon magnetoplasmic waves. The magnetic flows are [2]

$$\phi_x = \int_{-a}^{a}H_x dz = H\left(\frac{tgk_- a}{k_-} + \frac{tgk_+ a}{k_+}\right),$$
$$\phi_y = \int_{-a}^{a}H_y dz = iH\left(\frac{tgk_+ a}{k_+} - \frac{tgk_- a}{k_-}\right). \tag{10}$$

In the case $\omega_H\tau >> 1$ we have from (9)

$$k_- = \sqrt{\frac{\omega_p^{\ 2}\omega}{\omega_H c^2}}\left(1 - \frac{i}{\omega_H\tau}\right) \tag{11}$$

From (10a) we can determine an effective magnetic permeability

$$\mu_{eff} = \frac{\phi_x}{2aH} = \frac{tgk_- a}{2k_- a} + \frac{tgk_+ a}{2k_+ a} \tag{12}$$

The μ_{eff} has a maximum in the case of resonance

$$(\mathrm{Re}\,k_-)a = \frac{n\pi}{2},\ n=1,3,5,\ldots \tag{13}$$

and is equal

$$\max\mu_{eff} = \frac{2i\omega_H\tau}{n^2\pi^2},\ n=1,3,5,\ldots \tag{14}$$

The impedance Z of the inductive coil is purely active

$$Z = i\omega L_0\mu_{eff} = \frac{2\omega_H\tau}{n^2\pi^2}\omega L_0, \tag{15}$$

where L_0 is the coil inductivity without the core.

By help of (11) and (13) we can rewrite the formula (15) in the form

$$Z = \omega_H\tau\frac{\omega_H c^2}{2\omega_p^{\ 2}a^2}L_0. \tag{16}$$

3. TWO TYPES OF CHARGE CARRIERS

If the semiconductor sample contains two types of electrons with different densities and mobilities the characteristic equation for magnetoplasmic wave vector $\underset{\rightarrow}{k}$ can be written as follows [4]

$$k_\pm^{\ 2} = \omega\mu_0 eN_1u_1\left(\frac{-i\pm u_1B}{1+(u_1B)^2}\right) + \omega\mu_0 eN_2u_2\left(\frac{-i\pm u_2B}{1+(u_2B)^2}\right) \tag{17}$$

where N_1, N_2 and u_1, u_2 are the densities and mobilities of the different types of charge carriers; μ_0 - magnetic permeability of vacuum.

The resonant curves in dependence of magnetic field $\underset{\rightarrow}{B}$ were calculated from (17) for various values of N_1, N_2, u_1, u_2, and angular frequencies ω, and are shown on Fig.1-3.

The resonant curves of amplitude and phase for forward wave *K* and reflected wave *P* from InSb plate in dependence of magnetic field *B* are calculated for frequency f=300 MHz. Parameters of the semiconductor plate: thickness a=5 mm, dielectric constant ε=15, charge carrier density $N = 0{,}15*10^{23} m^{-3}$, charge carrier mobility $u = 5m^2V^{-1}s^{-1}$.

Calculations are made using program [3].

The first main peak for $B_1 = 29T$ corresponds to the resonance of electrons with higher mobility u_1 (density N_1). The second peak for $B_2 = 7T$ is responsible for the resonance of both carrier types with summary density ($N_1 + N_2$). The heights of both peaks are proportional to the products ($u_1 B$) and ($u_2 B$) respectively. The comparison of experimental and theoretical resonant curves provides the possibility to calculate the parameters N_1, N_2, u_1, and u_2 for both types of charge carriers.

Resonant curves in Fig.1 are calculated for one type and in Fig.2 for both types of charge carriers, with different sign of carrier densities u_1 and u_2. In comparison, resonant curves in Fig.1 and Fig.2 showed, that different sign of mobilities u_1 and u_2 increases second and higher resonance maximums in resonant curves.

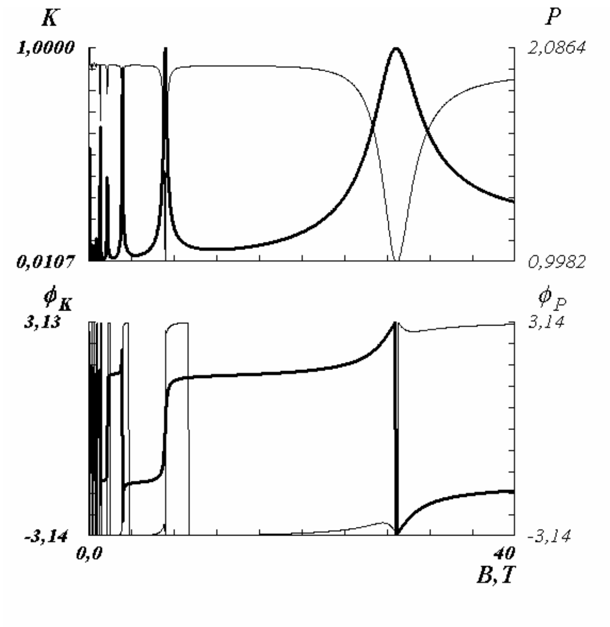

Fig.2 The resonant curves in dependence of magnetic field $\underrightarrow{B}$ for

$$u_1 = -5m^2V^{-1}s^{-1}, u_2 = 5m^2V^{-1}s^{-1}$$
$$N_1 = N_2 = 0{,}15*10^{23} m^{-3}$$

Fig.3 illustrates influence of semiconductor parameters with different sign of mobilities u_1 and u_2 for two types of charge.

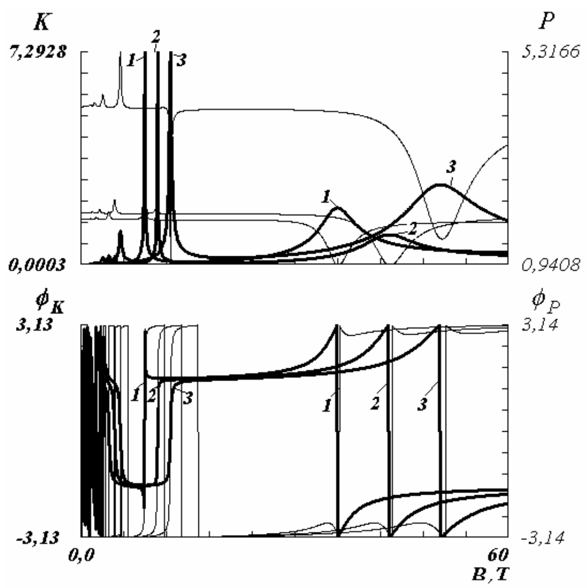

Fig.1 The resonant curves in dependence of magnetic field $\underrightarrow{B}$ for

$$u_1 = u_2 = 5m^2V^{-1}s^{-1},$$
$$N_1 = N_2 = 0{,}15*10^{23} m^{-3}$$

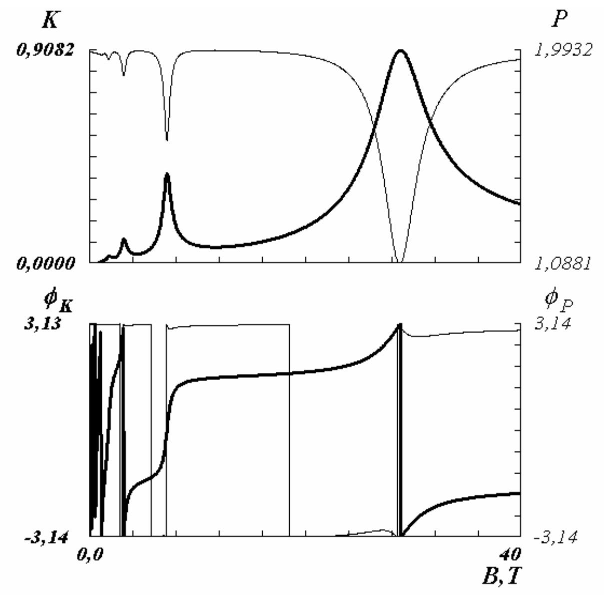

Fig.3 The resonant curves in dependence of magnetic field $\underrightarrow{B}$ for $u_1 = -2.5*m^2V^{-1}s^{-1}, u_2 = 5m^2V^{-1}s^{-1}$

$$N_1 = 0{,}3*10^{23} m^{-3},\ N_2 = 0{,}15*10^{23} m^{-3},$$

1 – f=250 MHz, K_{max}=4,01, 2 - f=300 MHz, K_{max} =7,19, 3 - f=350 MHz, K_{max} =2,74

We can see that pick various combinations of parameters influence resonant curves. Different values of carrier densities N_1, N_2 don't influence the resonant curves. Resonant curves with increasing frequency marginally move to low frequency range.

The additional peaks on Fig. 3 for small fields (<B) pertain to the higher resonant modes n>1.

There are illustrated increasing maximums for second resonance of resonant curves for forward wave more than 1.

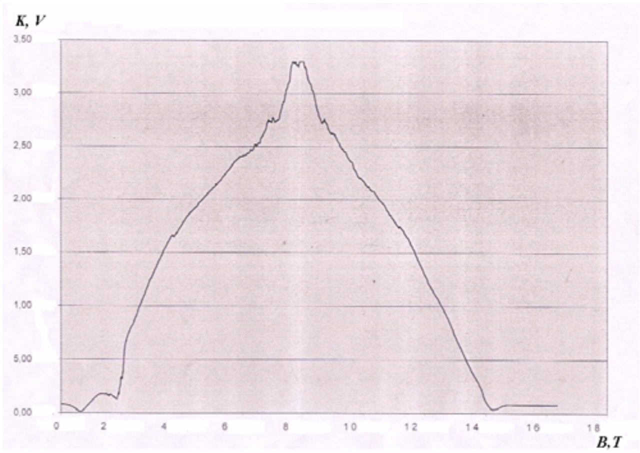

Fig.4 The experimental resonant curve in dependence of magnetic field $\underrightarrow{B}$

4. EXPERIMENTAL RESULTS

The theoretical results were confirmed experimentally for the n-Ge semiconductor material. In Fig.4 the experimental resonance curve is shown for the case f=25 MHz and specimen thickness d=3 mm. The relatively weak signal in detection coil for B=1,5 T pertains to the electrons with a higher mobility 0,65 $m^2V^{-1}s^{-1}$, and the stronger maximum for B=8 T – to the electrons with a smaller mobility 0,125 $m^2V^{-1}s^{-1}$.

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5. CONCLUSIONS

The measurement of the charge densities and mobilities for the charge carriers of various types in semiconductor materials by help of magnetoplasmic waves can be provided in contactless mode. Many semiconductors thus can be investigated if the high magnetic fields (~30 Tesla) are available. The measurement results are in compliance with the data obtained by the use of already existing methods.

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