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Accuracy of the measurement with the second order axial gradiometer

I. Šimáček, P. Jurdák, J. Maňka and M. Škrátek

Department of Magnetometry, Institute of Measurement Science, Slovak Academy of Sciences, Dúbravská cesta 9, 841 04 Bratislava, Slovakia, e-mail: umersima@savba.sk

Determining the accuracy of the measurement by using the second order axial gradiometer is presented. Output signals from the SQUID gradiometer system and calculated values of systematic errors depending on the shape and positioning of small cylindrical samples containing magnetized material are introduced. The measurement of the gradiometer detection characteristics and systematic error analysis were accomplished.

Keywords: second order axial gradiometer, magnetic measurement, cylindrical sample, systematic error

1. INTRODUCTION

SPECIFIC PROPERTIES of gradiometer systems are well known and their advantages are often employed in biomagnetic measurements. One possible area involves research on biological materials in a human body containing various paramagnetic or ferri- alternatively ferromagnetic substances. Using the small cylindrical samples, magnetic properties or concentrations of these substances are frequently investigated. With the help of various SQUID systems with the 2nd order axial gradiometers, the remanent magnetic induction B_r generated by the magnetized sample content is measured [1]. It is known that detection characteristics [2] of that type of gradiometer are relatively narrow, consequently the acquired output data strongly depend on the sample dimensions and its position to the gradiometer. Technical arrangement for the measurement of the gradiometer detection characteristics and calculation of some systematic errors in magnetic measurement of samples with volume up to 12 cm^3 are presented. The aim of this work was to investigate the accuracy of the measured data.

2. SUBJECT & METHODS

To evaluate the influence of the shape, volume and position of a measured sample on the accuracy of the observed output data, the following procedure has been made:

i. The powdered γ-Fe_2O_3 with the mass of 5 mg was inserted into the small cylindrical hole (diameter d_1 = 1 mm, volume $V_1 \approx 1$ mm^3), which had been bored in a hardened epoxy matrix.

ii. This miniature sample S_1 was magnetized for 30 s in the steady homogeneous magnetic field of 120 mT, whereby S_1 achieved a remanent magnetic moment $\vec{m}_{r1}(0, 0, m_{r1})$.

iii. Then S_1 was positioned in stages on a movable platform at five vertical distances h from the centre of the closest coil of the 2nd order axial gradiometer (baseline length b_g = 4.5 cm, with d_g = 3 cm diameter coils). Consequently, $\vec{m}_{r1}$ was oriented along the z-axis normal to the planes of the gradiometer pick-up coils, Fig.1.

iv. In every position h, S_1 was shifted from the midpoint horizontally at the distances of $x = \pm$ 15 cm. During the whole movement the output voltage U_1 was recorded by the one-channel SQUID gradiometer system (white noise level < 30 fT Hz$^{-1/2}$)

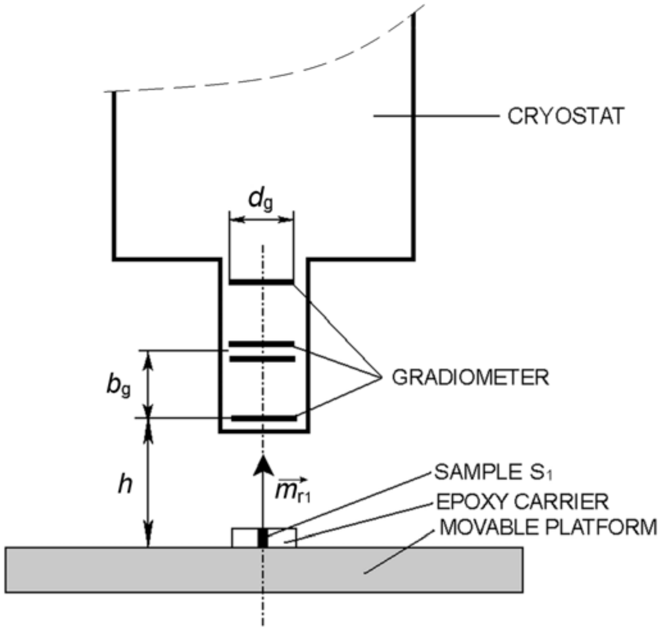

Fig.1 Schematic arrangement for magnetic measurement of the sample S_1 with the 2nd order axial gradiometer.

Measured values of U_1 were proportional to z component of the remanent magnetic induction B_r from the magnetized S_1. Under assumption that this magnetized mass element represents one magnetic dipole, the gradiometer detection characteristics $U_1 = f(m_{r1}, x, h)$ could be approximately determined.

3. RESULTS

Fig.2 shows the output signals U_1 induced by horizontal positioning of S_1 as a function of distances h = 3, 4, 5, 7 and 9 cm. It is evident that each value of U_1 is smaller in comparison to U_{1max} being measured in the central position of $x_0 = 0$. This difference U_{1max} - U_1 can be defined as a systematic error $e_1 = \Delta U_1$ related to the given gradiometer construction, horizontal position x and applied vertical position h of S_1.

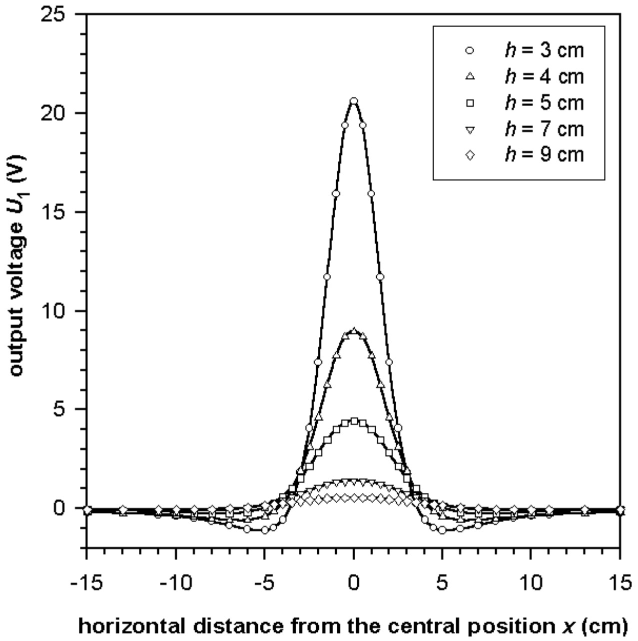

Fig.2 Experimental gradiometer detection characteristics in response to horizontal position x of the sample S_1 being placed at the vertical distances h.

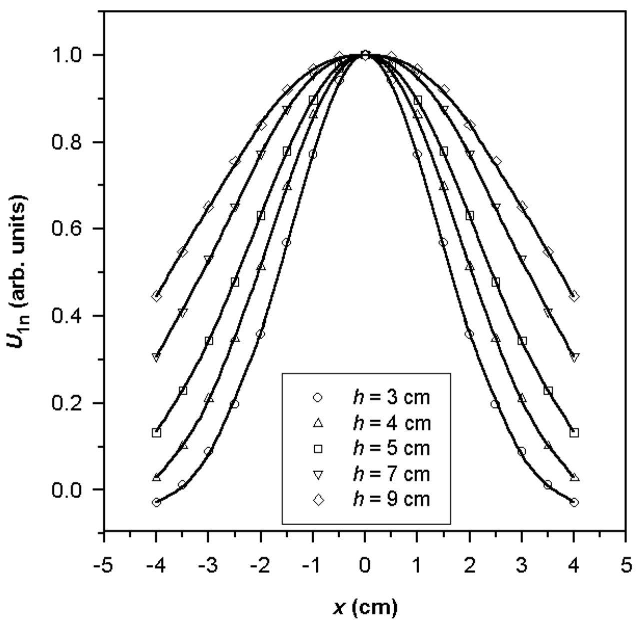

Fig.3 Normalized gradiometer detection characteristics in response to the horizontal position x for various h.

The values of U_1 were normalized with respect to U_{1max} and then fitted to U_{1n} by the equation

$$U_{1n} = y_0 + a\exp(-0.5\left(\frac{x - x_0}{b}\right)^c) \tag{1}$$

where a, b, c, x_0 and y_0 are the parameters of the used fitting. The normalized gradiometer detection characteristics U_{1n} are plotted in Fig.3.

The first step was aimed at estimating the systematic errors e_s for real larger cylindrical samples S with the height of 1 mm and the diameters of d = 1, 2, 3 and 4 cm, being gradually placed at the vertical distances of h = 3, 4, 5, 7 and 9 cm. Assuming that the sample S is magnetically isotropic and consists of many S_1 without interactions among them, it can be divided into the set of rings with the widths of 1 mm, Fig.4.

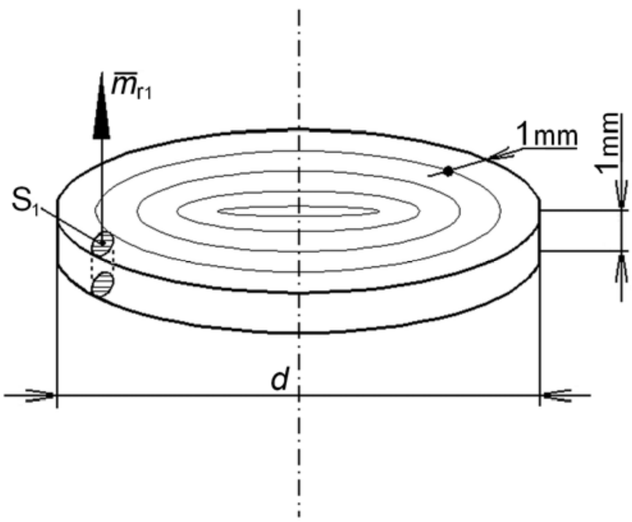

Fig.4 Schematic configuration of the sample S_1 divided into the set of rings.

Then the magnitude of the measured total output voltage U_s will be dependent both on the position of each S_1 and on the magnitude of the area of the ring which S_1 lies on. In the first approximation, e_s can be determined (in percents) by the following equation

$$e_s = 100 - \sum_{1}^{m} P_m U_{1mn} \tag{2}$$

where m is the number of rings with the width of 1 mm, P_m is the ratio of the m-ring area to the whole sample's area P_c (also in percents) and U_{1mn} is the normalized output voltage derived from (1). Figure 5 shows the corresponding calculation of e_s for the cylindrical sample S with the thickness of 1 mm for four diameters d (in cm). Data were fitted with an exponential decay function

$$e_s = y_0 + a\exp(-bh) \tag{3}$$

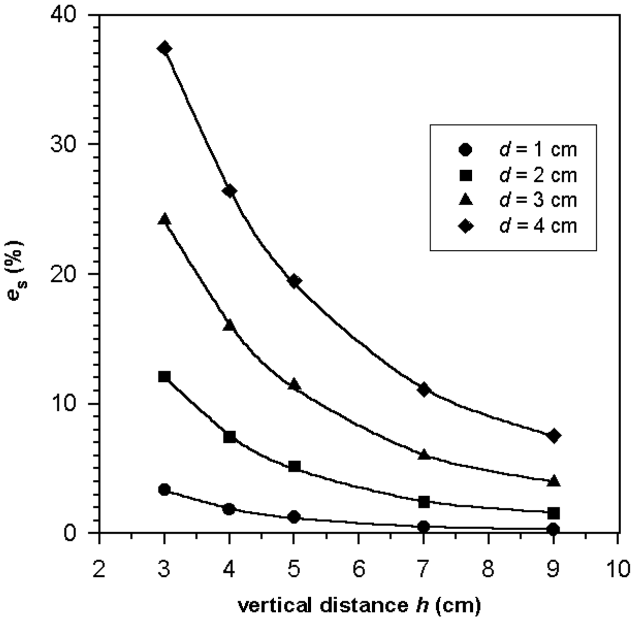

Fig.5 The calculated dependences of e_s on the vertical distance h for samples S having several diameters d.

The resulting e_s expresses how much (in percents) of the measured output voltage U_s falls off in comparison to the output voltage U_{sm} measured if all $\vec{m}_{r1}$ of the sample S were theoretically concentrated in the midpoint $x_0 = 0$. Consequently, the wider the sample and the closer to the gradiometer, the greater e_s is achieved. It can be shown in this example: If S with d = 4 cm changes its position from h = 4 cm to 9 cm, e_s decreases approximately five times, specifically, from 37.46% to 7.45%. Likewise, if S has only d = 1 cm e_s also decreases from 3.35% to 0.30% after the same change of h.

In practice, the samples have various heights. Therefore, the total systematic error e_{sc} for cylindrical samples S_c with heights y ranging from 1 to 10 mm was calculated. Using the modified (3) the systematic errors e_{sy} for each 1 mm layer of S_c were determined by the following formula

$$e_{sy} = y_0 + a\exp(-b(h - 0.1(s-1))) \tag{4}$$

where s is the appropriate 1 mm layer measured from the base of the S_c. Then, the final e_{sc} magnitude can be expressed as the sum of e_{sy} by

$$e_{sc} = \frac{\sum_{s=1}^{s_c} e_{sy}}{s_c} \tag{5}$$

where s_c is the number of 1mm layers in the S_c.

The importance of the sample shape for the e_{sc} magnitude is shown in Fig. 6 illustrating e_{sc} as a function of y for two samples S_{c1} and S_{c2}. These samples have the same volume of $V = 1.26$ cm^3 and contain the same magnetic material but they have different shapes. The first, $S_{c1,}$ has dimensions of d_1 = 4 cm and y = 1 mm and the second one, $S_{c2,}$ is with the dimensions d_2 =2 cm and y = 4 mm. In spite of their identical magnetization $M \sim V$, in case of S_{c1} at the distance of h = 4 cm the calculated e_{sc} is 26.6%, in comparison to e_{sc} = 7.64% for S_{c2}. These two identical samples at h = 9 cm resulted in e_{sc} = 7.72% in the first case and e_{sc} = 1.65% in the second one.

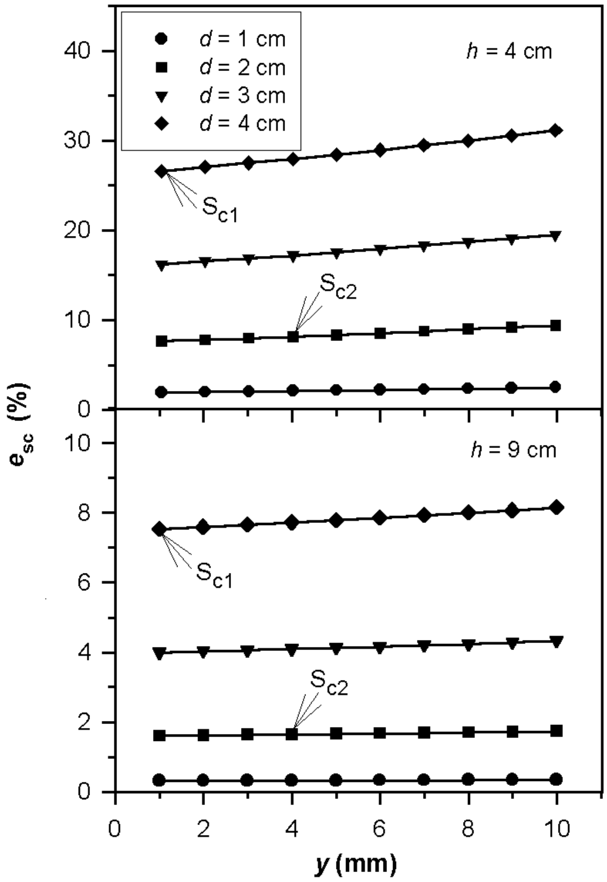

Fig.6 Calculated values of e_{sc} related to the height y at the distances h = 4 cm and 9 cm for four diameters d of the samples S_c.

The effect of various y on e_{sc} may be also demonstrated by the following example: if dimensions of S_c are d = 1 cm, y = 1 mm and this sample is located at h = 4 cm, then the calculated e_{sc} is 1.94%. By increasing y up to 10 mm, e_{sc} increases only to 2.5%. For larger S_c with dimensions of d = 4 cm and y = 1 mm, e_{sc} rises from 26.60% to 31.11% if y enlarges from 1 mm to 10 mm. Taking into account the sensitivity of the measuring system in terms of accuracy, the samples should be as small as possible, with the shape that is rather higher than wider and placed under the sensor as far as possible.

Of course, the accuracy of the measurement is also affected by the gradiometer construction. From this point of view, e_{sc} decreases if the gradiometer detection characteristics are relatively wide. For example, it can be influenced by increasing of the base length b_g. Assuming that the diameters of the gradiometer coils are smaller than the distance h, then the relation of B_r between the position of measured magnetic particle can be approximately determined by the following relation [3].

$$\vec{B}_r(\vec{m},\vec{r}) = \frac{\mu_0}{4\pi}\left[\frac{3(\vec{m}.\vec{r})\vec{r}}{r^5} - \frac{\vec{m}}{r^3}\right] \tag{6}$$

where $\vec{B}_r(\vec{m},\vec{r})$ is the magnetic induction in the individual gradiometer coils generated from the magnetized particle with the magnetic dipole moment $\vec{m}(0,0,m)$, and $\vec{r}$ is the position vector of that magnetic dipole to the sensing and compensating gradiometer coils. Using (6), Fig.7 illustrates the resulting dependences of $B_{rn}(x)$ normalized to the largest value, for all defined positions of coils when the base lengths b_g are 2.5, 4.5 or 8 cm. One can see that the most flat course has the gradiometer detection characteristic $B_{rn}(x)$ for b_g = 8 cm. It means, that the greater b_g, the smaller e_{sc} regardless the sample dimensions or its distance h from the gradiometer.

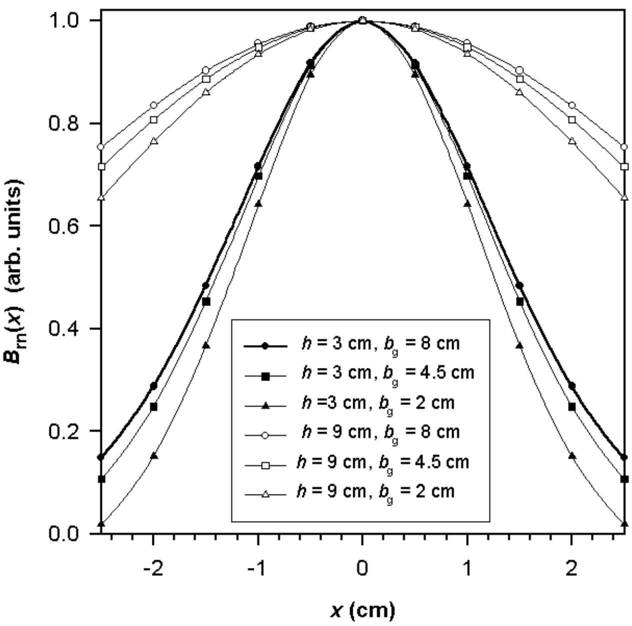

Fig.7 Calculated values of e_{sc} related to the height y at the distances h = 4 cm and 9 cm for four diameters d of the samples S_c.

As shown above, in the measurements of the small-volume samples using 2nd order axial gradiometers it is important to take into account the shape, dimensions and placement of the measured sample. In practice, it is necessary to do some compromises among the used sensitivity in the measuring process, the degree of the noise reduction and the accuracy of the measurement. Thus, the systematic errors should be corrected or have to be taken into account while estimating the standard uncertainty of the used measuring method.

4. CONCLUSION

We have presented a type of technique and calculation for assessing the accuracy of the measuring procedure for the given axial gradiometer system. The systematic errors were determined on the basis of the measured detection characteristics of one specific 2nd order axial gradiometer. The measured and calculated data showed that the accuracy strongly depended on the shape and size of the samples and on their position to the gradiometer system.

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