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Modelling and Simulation of the Pyroelectric Detector Using MATLAB/Simulink

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An attempt was made to obtain a simple Laplace transfer function model of a pyroelectric detector which would make an excellent basis for the analysis of its dynamic behaviour. Knowledge of the transfer function model of the pyroelectric detector is necessary for its simulation studies in MATLAB/Simulink environment. A simple model of the pyroelectric detector was implemented in MATLAB/Simulink and some exemplary results of investigation of its voltage response to incident radiation and frequency characteristics were presented.

Keywords: modelling of pyroelectric detector, transfer function model of pyroelectric detector, simulation of pyroelectric detector

1. INTRODUCTION

PYROELECTRIC DETECTORS, thanks to their specific spectral properties, have found a wide range of applications, e.g. in thermovision, remote temperature measurement, monitoring and measurements of laser emission parameters, and military and civil security systems. In contrast to other types of thermal detectors the pyroelectric ones respond to excitation produced by radiation signal of variable power. The use of such detectors for monitoring and measurement of sources of a constant radiation power, e.g. continuous wave lasers (CW), is possible under the condition that their radiation is modulated by electromechanical or electrooptical modulators. Theoretical description of the voltage or current response to radiation absorbed by the detector can be made by analytical expressions and an equivalent circuit diagram of a pyroelectric detector was proposed already in the 1960s and 1970s. Mathematical description of the pyroelectric detector response can be a difficult task, especially when the radiation wave incident onto the active surface of this detector has a shape other than sinusoidal. The problem can be solved by computer simulation methods.

It should be noted that, in general, simulation methods have been rarely applied for investigation of pyroelectric detectors despite the fact that contemporary computer methods offer many tools for such investigation. The subject has been treated in details only in a few papers. For example in [1] the authors present an interesting model of a pyroelectric sensor of infrared radiation whose thermal and electric properties are modelled by equivalent circuit diagram and simulations carried out with the help of the Tina program. The papers [2-3] present a model of the pyroelectric detector studied in the SPICE environment. It is worth to note that in some other papers only a short information is given about the use of an unspecified computer program for getting graphic illustration of response of a pyroelectric detector.

In this paper, implementation of a Laplace transfer function model of uncoated (front surface electrode is a thin layer of metal) pyroelectric detector in MATLAB/Simulink environment is presented to enable performance of simulation studies of the detector response to different types of radiation signals.

Literature survey has shown no publications on the use of such model of pyroelectric detector and its studies by simulations in the MATLAB/Simulink environment. In the opinion of the author this environment is highly suitable for the realisation of simulation study of a pyroelectric detector on condition of the use of the correct Laplace transfer function of this detector model.

2. LAPLACE TRANSFER FUNCTION MODEL OF THE PYROELECTRIC DETECTOR

A pyroelectric detector is essentially a capacitor with capacitance C_d. The dielectric of this capacitor is a thin film made of pyroelectric material of the thickness d and surface A coated with metallised film. If the radiation of the power $\Phi(t)$ varying in time is incident on the active surface of the pyroelectric detector, the electric charge $q(t)$ appears on the electrodes of this detector as a result of thermal and electric processes in the pyroelectric material. The signals with information content on the radiation signal can be voltage $V(t)$ on the detector electrodes or current $I_p(t)$ flowing through the low load resistance of the detector output. The two alternative modes of work of the pyroelectric detector provide voltage or current output signals. They are referred to as the voltage mode in which the detector cooperates with an amplifier of high input resistance R_L and current mode [4] in which the detector cooperates with a trans-impedance amplifier with very small input resistance.

The process of conversion of the radiation energy into electric signal is composed of three stages. The first is the thermal conversion of the radiation stream of the power $\Phi(t)$ incident on the detector surface into change of the detector temperature $T(t)$, the second is the thermal/ electric conversion of the temperature changes $T(t)$ into electric current of a current source $I_p(t)$ and the third is the conversion of the current signal $I_p(t)$ into a voltage signal $V(t)$. In order to make a mathematical description of the pyroelectric detector it is necessary to analyse the thermal and electric equivalent circuit of the detector. A good mathematical model and equivalent circuit diagram of a pyroelectric detector commonly used up to now in research work [5-7] was proposed in the 1960s and 1970s [8, 9]. This model is correct assuming that the pyroelectric material film is sufficiently thin, so that temperature changes caused by the irradiation absorbed are the same in the whole volume of this film [9].

The equivalent circuit diagram [5-8] for the detector working in the voltage mode illustrating the three stages of conversion is given in Fig.1.

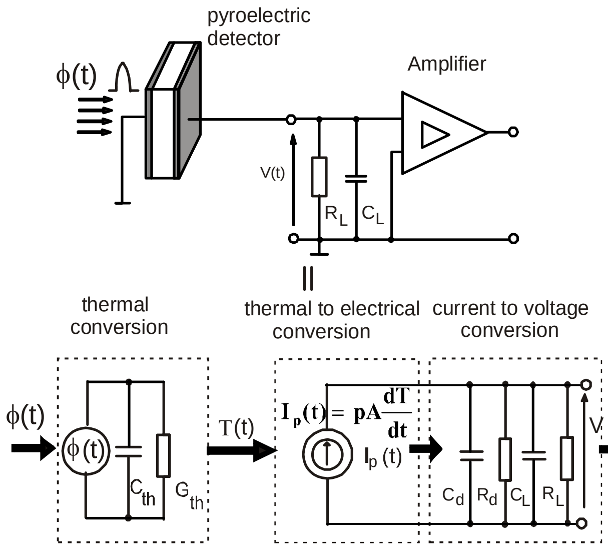

Fig.1 . Thermal and electrical equivalent circuit of the pyroelectric detector and input amplifier

The equivalent circuit includes three blocks presenting earlier-mentioned stages of signal conversion for the pyroelectric detector - thermal conversion, thermal to electrical conversion, and current to voltage conversion described by equations (1), (2) and (3) respectively.

$$C_{th}\frac{dT(t)}{dt}+G_{th}T(t)=\eta\Phi(t)\,, \qquad (1)$$

$$I_p=pA\frac{dT(t)}{dt}\,, \qquad (2)$$

$$C\frac{dV(t)}{dt}+\frac{V(t)}{R}=I_p(t)\,, \qquad (3)$$

where: C_{th} – thermal capacity of pyroelectric detector,
G_{th} - thermal conductance of pyroelectric detector,
η – absorption coefficient of radiation,
p – pyroelectric coefficient,
C – equivalent capacitance for parallel connected pyroelectric capacitance C_d and input amplifier capacitance C_L: $C = C_d + C_L$,
R - equivalent resistance for parallel connected leakage resistance R_d of pyroelectric detector and input amplifier resistance: $R = R_d R_L/(R_d + R_L)$,

Each of the conversion stages shown in Fig.1 and described by differential equations (1-3) can be modelled by a Laplace transfer function defined as the ratio of the Laplace transform of the output signal to the Laplace transform of the input signal under the assumption that all initial conditions are zero.

- Transfer function $G_T(s)$ for thermal conversion

As a result of Laplace transform of differential equation (1), the transfer function $G_T(s)$ of the block responsible for realization of thermal conversion is described by

$$G_T(s)=\frac{T(s)}{\Phi(s)}=\frac{\eta}{sC_{th}+G_{th}}\,. \qquad (4)$$

To carry out the simulation study it is recommendable to take into account such parameters of the detector as the thermal constant τ_{th}, active area A and volume specific heat c' in the expression for the transfer function $G_T(s)$. Taking into account the mathematical expressions describing the thermal time constant $\tau_{th} = C_{th} / G_{th}$ and thermal capacity of the detector $C_{th} = c'Ad$, the transfer function $G_T(s)$ (eq. 4) can be rewritten into an alternative form

$$G_T(s)=\frac{\eta\tau_{th}}{c'dA(s\tau_{th}+1)}\,. \qquad (5)$$

- Transfer function $G_{TIp}(s)$ for thermo-electrical conversion

As a result of Laplace transform of equation (2) one can obtain the transfer function $G_{TIp}(s)$ given by

$$.\ G_{TIp}(s)=\frac{I_p(s)}{T(s)}=spA \qquad (6)$$

- Transfer function $G_{IpV}(s)$ for current-voltage conversion

Taking Laplace transform of equation (3) one can obtain the transfer function $G_{TIp}(s)$ for current-voltage conversion

$$G_{IpV}(s)=\frac{V(s)}{I_p(s)}=\frac{R}{sRC+1}=\frac{R}{s\tau_e+1}\,, \qquad (7)$$

where τ_e – electrical time constant of equivalent circuit of the pyroelectric detector and amplifier ($\tau_e = RC$).

Taking into account the procedures of creating block diagrams well known from the automatic control theory, the transfer function model of the pyroelectric detector can be introduced as a series connection of three blocks with transfer functions $G_T(s)$, $G_{TIp}(s)$ and $G_{IpV}(s)$ describing properties of the appropriate signal conversion stage..

In general, the most important result of simulation study is the information on the relation between the voltage response $V(t)$ of the detector and the input signal that is the radiation power $\Phi(t)$ absorbed. This relation can be obtained by determining the equivalent transfer function $G(s)$ describing dynamical properties of the pyroelectric detector

$$G(s) = G_T(s) G_{TIp}(s) G_{IpV}(s) \; . \tag{8}$$

After having inserted into eq. (8) earlier derived relations (5), (6) and (7) describing the transfer functions $G_T(s)$, $G_{TIp}(s)$ and $G_{IpV}(s)$ we get the transfer function $G(s)$ of the pyroelectric detector

$$G(s) = \frac{p\eta R}{c'd} \frac{s\tau_{th}}{(s\tau_{th}+1)(s\tau_e+1)} \; . \tag{9}$$

Taking into account other known relations describing the electric time constant $\tau_e = RC$ and the capacity of the capacitor $C = \varepsilon A / d$, and after some transformations, we can arrive at the alternative form of the transfer function $G(s)$ of the pyroelectric detector

$$G(s) = \frac{p\eta}{c'dC} \frac{s\tau_{th}\tau_e}{(s\tau_{th}+1)(s\tau_e+1)} \; . \tag{10}$$

In Fig.2, the transfer function model of the pyroelectric detector is shown.

$$\phi(t),\ \phi(s) \rightarrow \boxed{G(s) = \frac{p\eta}{c'dC} \frac{s\tau_{th}\tau_e}{(s\tau_{th}+1)(s\tau_e+1)}} \rightarrow V(t),\ V(s)$$

Fig.2. Transfer function model of the pyroelectric detector

3. Simulation study of the pyroelectric detector in the MATLAB-Simulink environment

Simulation study requires the knowledge of numerical values of the detector parameters needed for the mathematical form of the transfer function model of the pyroelectric detector. The exemplary simulation study presented below was carried out for the prototype of the polyvinylidene fluoride (PVDF) pyroelectric detector designed and constructed by the author. Construction and some results of experimental studies performed with this detector have been presented among others in [10-12]. Table 1 gives the parameters of the detector needed for the simulation study.

Table 1. Parameters of the PVDF pyroelectric detector

Name	Value	Unit
pyroelectric coefficient p	$30\cdot10^{-6}$	C/cm^2K
volume specific heat c'	2.4	J/cm^3K
permittivity ε	$106\cdot10^{-14}$	F/cm
pyroelectric film thickness d	25	µm
thermal conductivity g_{th}	0.00135	W/cmK
detector active surface A	$132\cdot10^{-2}$	cm^2
pyroelectric detector capacitance C_d	560	pF
amplifier input capacitance C_L	negligible	pF
amplifier input resistance R_L	10	MΩ
electrical time constant of detector--amplifier circuit τ_e	0.0056	s
thermal time constant τ_{th}	0.011	s
absorption coefficient of radiation η	1	-

Below, a few selected examples are given to illustrate the method of simulation study of the pyroelectric detector in the MATLAB/Simulink environment.

- Frequency response of the voltage responsivity of the pyroelectric detector

Frequency response of the pyroelectric detector voltage signal to sinusoidal modulated radiation is in fact the well known Bode magnitude plot which can be easily performed in MATLAB. Particularly interesting is the family of Bode plots for various parameters, e.g. for various thicknesses of the pyroelectric film.

To obtain the Bode plots in MATLAB, the transfer function of the detector expressed by eq. (10) should be transformed to the form with polynomials in the numerator and denominator

$$G(s) = \frac{\frac{p\eta}{c'dC}\tau_{th}\tau_e s}{s^2\tau_{th}\tau_e + s(\tau_{th}+\tau_e)+1} \; . \tag{11}$$

An exemplary simulation shows the dependence of the voltage responsivity R_V on the polymer thickness: $d_1 = 25$ µm, $d_2 = 50$ µm i $d_3 = 100$ µm. Worth to note is that the change of polymer value thickness d influences both the electrical and thermal time constant values according to dependencies $\tau_e = R\,\varepsilon A/d$ and $\tau_{th} = c'd^2/g_{th}$. As a result of the calculations, the transfer functions for each of the above-mentioned pyroelectric film thickness values are described:

- $G_1(s) = 0.055s / (0.0000616s^2 + 0.0166s + 1)$ for $d_1 = 25$ µm,
- $G_2(s) = 0.11s / (0.000124s^2 + 0.0468s + 1)$ for $d_2 = 50$ µm,
- $G_3(s) = 0.22s / (0.000246s^2 + 0.177s + 1)$ for $d_3 = 100$ µm.

The MATLAB scripts created for the simulation of Bode plot are listed below:

```
>> SYS1=TF([0.055 0], [0.0000616 0.0166 1])
Transfer function:
        0.055 s
-----------------------------------
6.16e-005 s^2 + 0.0166 s + 1
>> SYS2=TF([0.11 0], [0.000124 0.0468 1])
Transfer function:
        0.11 s
-----------------------------------
```

```
0.000124 s^2 + 0.0468 s + 1
>> SYS3=TF([0.22 0], [0.000246 0.177 1])
Transfer function:
      0.22 s
-----------------------------------
0.000246 s^2 + 0.177 s + 1
>> Bode (SYS1, SYS2, SYS3)
```

Fig.3 presents the Bode plots of the pyroelectric detector for various thicknesses of the pyroelectric PVDF film.

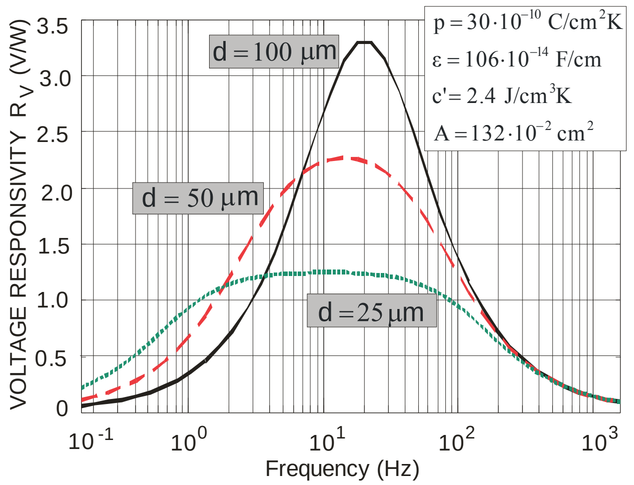

Fig.3. Voltage responsivity of a pyroelectric detector with different thickness *d* of the pyroelectric PVDF film. Other parameters of the pyroelectric detector are listed in table 1

Analysis of the curves presented in Fig.3 has revealed that in the intermediate range of frequencies, near the maximum voltage responsivity R_v, the voltage responsivity of the detector increases with decreasing thickness of the pyroelectric film. This behaviour of the detector has been confirmed by experimental results reported among others in [13]. However, for small frequencies of modulation of the radiation signal (below 2 Hz), the dependence of voltage responsivity on the pyroelectric film thickness is the opposite; that is, the voltage responsivity increases with increasing thickness of the pyroelectric film, as follows from the simulation results shown in Fig.3. It should be noted that the above-described behaviour of the detector has not been reported in the available literature.

With the use of a collection of MATLAB scripts similar to those mentioned above, it is possible to get families of Bode plots practically for any parameter of the detector. Fig.4 presents the less known dependence of the pyroelectric detector voltage responsivity on frequency with the thermal time constant as parameter.

As follows from the plots presented in Fig.4, with increasing thermal time constant the voltage responsivity of the detector increases. This is a desirable effect in practical applications. However, on the other hand, increase in the thermal time constant leads to undesirable changes in the frequency dependence of voltage responsivity R_V, as its maximum shifts towards lower frequencies.

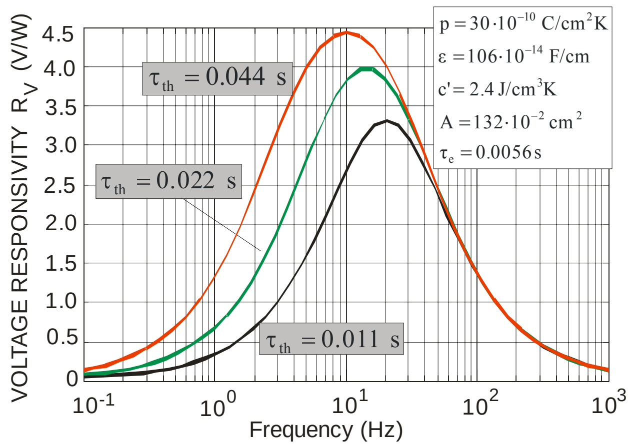

Fig.4. Voltage responsivity of the pyroelectric detector obtained with various thermal time constants τ_{th} of the pyroelectric detector. Other parameters of the pyroelectric detector are listed in table 1

- Voltage response of the pyroelectric detector to different shapes of periodically modulated radiation

The Laplace transfer function model $G(s)$ of the pyroelectric detector described by eq. (10) can be easily implemented in Simulink. For simulation study it is convenient to express the transfer function model $G(s)$ in the form of a product of three blocks connected in series and characterised by transmittances of $G_1(s) = p\eta / c'dC$ $G_2(s) = \tau_{th} / (s\tau_{th} + 1)$ and $G_3(s) = \tau_e / (s\tau_e + 1)$. It is not a necessary operation but it permits minimisation of errors on modification of the detector parameters and easier prediction of consequences of these changes. Fig. 5 shows the simulation diagram which is equivalent to the above-mentioned mathematical model of the pyroelectric detector. Fig. 6(a-b) presents the voltage responses of the PVDF detector to the radiation $\Phi(t)$ of sinusoidal, rectangular (square wave) and trapezoidal shape with the same amplitude of the radiation power Φ = 1 W.

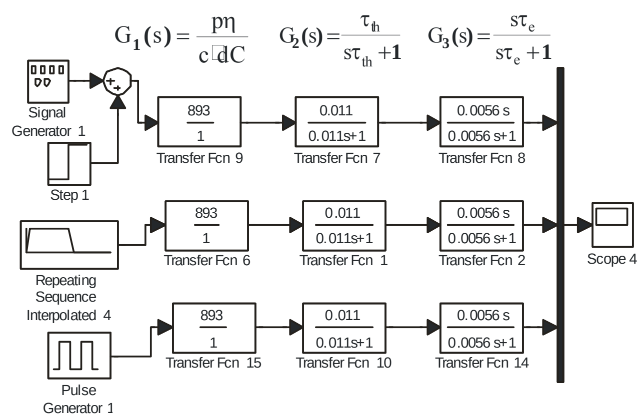

Fig.5. Simulink block diagram for simulation of voltage response of the pyroelectric detector

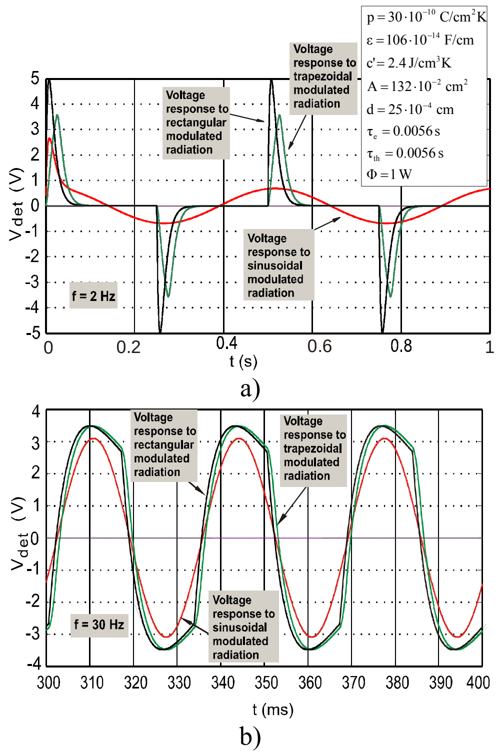

Fig. 6.Voltage response V_{det} of pyroelectric detector to sinusoidal, rectangular and trapezoidal modulated irradiation of power amplitude Φ = 1 W and of frequency a) f = 2 Hz, b) f = 30 Hz. Other parameters of the pyroelectric detector are listed in Table 1

Simulation results have shown that the voltage responsivity of the pyroelectric detector exposed to rectangular modulated radiation of low frequency is higher than that to sinusoidal and trapezoidal modulated radiation. When the frequency is relatively high (above 30 Hz) the voltage responsivity of the pyroelectric detector is similar in value for all shapes of modulated radiation. This type of simulation investigations gives important information about the properties of the pyroelectric detector in such applications as remote temperature measurements, monitoring of radiation power or thermovision measurements.

4. CONCLUSIONS

A simple transfer function model useful for simulation studies of dynamical performance of the pyroelectric detector has been presented. The simple model of uncoated pyroelectric detector was implemented in MATLAB/Simulink and some exemplary results of investigation of its voltage response to input radiation and frequency characteristics were presented. The simulations were made for the parameters of the polymer pyroelectric detector designed and constructed by the author and described in [10-12]. Exemplary results of less known simulations illustrating the influence of such parameters as the pyroelectric material film thickness and the thermal time constant on the responsivity of the detector are presented. The possibility of the simulation study on the basis of time dependences of voltage pyroelectric response to modulated radiation with different shapes is indicated.

Simulation in MATLAB/Simulink of the designed model of the pyroelectric detector gives excellent possibilities of investigation that are difficult to realize by analytical methods. It is very important that the tests can be easily modified and the results are obtained quickly.

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