On Modelling of Maximum Electromagnetic Field in Electrically Large Enclosures

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Abstract: The maximum electromagnetic field formed in the electrically large enclosures for a given input power has always been the focus of electromagnetic compatibility issues such as radiation sensitivity and shielding effectiveness. To model the maximums in a simple manner, the electrically large enclosure can be regarded as a reverberation chamber (RC), thus the generalized extreme value (GEV) theory based framework is used for both undermoded and overmoded frequencies. Since the mechanical stirrer is not easy to be installed like that for RC, frequency stirring and mechanical stirring related configurations are discussed, and the corresponding results have confirmed the validity of frequency stirring with the estimate of the parameters in GEV distribution. As for the maximum field, a comparison has been made between GEV distribution and IEC 61000-4-21, and the corresponding results have also highlighted that the maximum field can be assessed by frequency stirring configuration, and by GEV distribution with a desired confidence.

Keywords: Maximum electromagnetic field, electromagnetic compatibility, electrically large enclosure, reverberation chamber, generalized extreme value distribution.

1. INTRODUCTION

With exploding applications of high-power and high-frequency electronics in different areas, e.g., ships, aircraft, and electric vehicles, the electromagnetic compatibility (EMC) related issues have attracted the interest of the EMC community. For high frequencies (i.e., high mode densities), the sensitive electronics inside metallic enclosures may be threatened by the potential risks from electromagnetic interference (EMI) with apertures, cables, or antennas. To solve EMC problems as well as to enable reliable operation of the sensitive electronics, it is necessary to have an a priori knowledge about the maximum field formed in these enclosures.

Fortunately, the well-established theory for reverberation chambers (RCs), or for the general case of electrically large enclosures (supporting several field modes at the lowest frequency of interest), makes it possible to model the maximum field statistically [1], and the corresponding frequency should be covered in the overmoded regime (the low frequency limit no less than a mode count of 60). Specifically, the Cartesian field components sampled from RC follow a Rayleigh distribution, and thus the power is exponentially distributed, the maximums can then be described by the extreme value statistics [2]. For a given RC with the same input power, however, the power received by antenna relies on the number of independent samples when using the extreme value theory; that is, the estimated maximum field will have no upper bound especially for the unconditional probability density functions (PDFs) [3]-[6]. Therefore, the extreme value statistics may not be applicable as the estimates are contrary to common sense physics. Regarding low frequencies (i.e., low mode densities), there is no universally accepted mathematical model that sufficiently describes the field distribution corresponding to the undermoded frequencies (or even close to the lowest usable frequency of RC).

From the perspective of the tail behaviors of the empirical distribution, the generalized extreme value (GEV) theory based framework can be used to model the maximum field directly. It is easy to demonstrate that the exponential distribution converges to the maximum domain of attraction (MDA) of GEV distribution with shape parameter \( k \rightarrow 0 \) (i.e., the Gumbel type) [7]. Moreover, a large number of measurements have revealed that the results sampled at undermoded, weakly- and highly-overmoded frequencies show a good fitting with GEV distribution of \( k > 0 \) (fat tail, the Fréchet type), \( k \rightarrow 0 \) and \( k < 0 \) (thin tail, the reverse Weibull type), respectively [3], [5], [8], [9]. Without any doubt, the resonance modes with increasing frequency provide a more reasonable explanation for tail behaviors of
these samples. At low frequencies, it is really difficult to capture the samples related to the resonances, resulting in a significant fat tail in the empirical distribution, while the scattering of electromagnetic waves in a chaotic manner makes the resonances easier to be sampled, accounting for the thin tail at high frequencies.

Of particular note is that the samples should be independent and identically distributed (i.i.d.) for GEV distribution, thus requiring the mode-stirred techniques to be carried out within electrically large enclosures, e.g., the mechanical stirrer in RCs. However, for arbitrary electrically large enclosures, e.g., below-deck compartments in ships, ammunition containers in bunkers, aircraft cabins and bays [10], [11], the mechanical stirrer is not easy to be installed like that for RCs. According to the solution of scalar Green’s function, fortunately, it has been verified that the frequency stirring (FS) also offers good field uniformity, being regarded as an alternative for mechanical stirring (MS) [12]. Fundamentally, FS can provide sufficient numbers of i.i.d. samples, which have been applied for electromagnetic shielding effectiveness (SE) evaluation in nested RCs using GEV distribution [13].

The aim of this work is therefore to model the maximum electromagnetic field (EMF) for arbitrary electrically large enclosures covering the undermoded and the overmoded frequency regimes. To simplify the testing procedure, a vector network analyzer (VNA) is used for FS, the i.i.d. samples are recorded to extract the maximums intended for electromagnetic field (EMF) for arbitrary electrically large enclosures, e.g., electrically large enclosures covering the undermoded and the overmoded frequency regimes. To simplify the testing procedure, a vector network analyzer (VNA) is used for FS, the i.i.d. samples are recorded to extract the maximums intended for assessing the unknown parameters in the empirical distribution. Therefore, when calculating high-order moments for ME, e.g., the 2nd moment for the Fréchet type as shown in Fig.1., there is a significant fat tail for the CDF; that is, the maximum field cannot be easily captured in the case of the sparse resonance modes. While for overmoded frequencies, especially the case of the reverse Weibull type, a sufficient number of modes will make the maximum field to be easily captured, and thus resulting in the thin tail in Fig.1.

To provide intuitive explanation for the tail behavior of GEV distribution and the maximum field, herein we show a simple case as depicted in Fig.1.

B. Parameter estimation method

In general, the moment estimation (ME) and the maximum likelihood estimation (MLE) are commonly used methods for assessing the unknown parameters in the empirical distribution. For the maximum field in RCs, the samples essentially have an uncertainty within 3 dB at overmoded frequencies. Therefore, when calculating high-order moments for ME, e.g., the 2nd moment for the Gumbel type with 2 unknown parameters, the 3rd moment for the Fréchet and the reverse Weibull type with 3 unknown parameters, the uncertainty will inevitably be amplified, resulting in large uncertainty for these estimated parameters, despite the same frequency. As for MLE, a sufficient number of i.i.d. samples are recorded to extract the maximums intended for assessing the unknown parameters in the empirical distribution.
maximums (generally $N \geq 50$) should be sampled to ensure that these parameters are asymptotically unbiased. However, as discussed in [15], due to the mode density as well as the efficiency of mode-stirred wall, it is really difficult to obtain enough independent samples, especially at low frequencies.

In view of these issues, the L-moments (L-Ms) method is used in this work, which can be regarded as an alternative for small samples in many applications [3], [13], [16]. For the set of $N$ maximums $[\xi_1, \xi_2, \ldots, \xi_N]$, we can define an ordered sample $x_1 \leq x_2 \leq \cdots \leq x_N$, where $x_1 = \min_{i \in N} \xi_i$, and $x_N = \max_{i \in N} \xi_i$. As per [3] and [16], we can estimate the L-Ms $l_1$, $l_3$, and $l_5$, namely

$$l_1 = b_0$$
$$l_2 = 2b_1 - b_0$$
$$l_3 = 6b_2 - 6b_1 + b_0$$

where $b_0$, $b_1$, and $b_2$ are the parameters derived by the ordered sample,

$$\begin{cases}
  b_0 = \frac{\sum_{i=1}^{N} x_i}{N} \\
  b_1 = \frac{\sum_{i=1}^{N} (i-1)x_i}{N(N-1)} \\
  b_2 = \frac{\sum_{i=1}^{N} (i-1)(i-2)x_i}{N(N-1)(N-2)}
\end{cases}$$

Then, the unknown parameters in GEV distribution are [3]

$$\hat{k} = -7.8592 - 2.5994z^2$$
$$\hat{s} = \frac{-l_2}{(1-z^2)\Gamma(1-k)}$$
$$\hat{m} = l_1 + \frac{\hat{s}}{\hat{k}} \left(1 - \Gamma(1 - \hat{k})\right)$$

with

$$z = \frac{2l_2}{3l_4 + l_3} \cdot \frac{\ln 2}{\ln 3}$$

where $\Gamma(\cdot)$ is the Gamma function. For the sake of brevity, we can also use the $lmom$ function defined in [17].

3. MEASUREMENT SETUP

The measurement was carried out in an aluminum enclosure with the interior dimensions of 0.493 m × 0.389 m × 0.294 m, and the theoretical fundamental resonance of about 491 MHz. The enclosure is equipped with a mechanical stirrer, rotating about a vertical axis within a cylindrical volume of 0.26 m height and 0.11 m diameter.

The test configuration is shown in Fig.2. A two-port VNA, model Rohde & Schwarz ZNB 20, is used together with a pair of 10 cm monopole antennas (considered as the transmit (Tx) and receive (Rx) antennas, respectively). Rx antenna was placed at the working volume of enclosure and pointed at the stirrer. To minimize the direct coupling, Tx and Rx antennas were placed mutually orthogonal to each other.

The frequency range was set from 500 MHz to 6 GHz with a frequency step of 100 kHz, while the stirrer stepped 48 positions; the port power of VNA was set to 0 dBm. For either position of mechanical stirrer, 55001 sets of $S_{21}$ data were sampled. To the best of our knowledge, the $S_{21}$ is oversampled with respect to 100 kHz, and thus it is inferred that most of the resonances can be recorded.

4. RESULTS AND DISCUSSION

A. GEV parameters estimated by FS and MS configurations

To provide an estimate of the parameters (i.e., $k$, $s$, and $m$) in (2), one should extract $N$ maximums from the measurements. For FS related configuration, the stirring bandwidth (BW) should be rather carefully selected. At low frequencies, there may be no mode excited in the stirring bandwidth, resulting in incorrect estimation of the parameter $k$ (more details are explained in [13]). Herein, BW is set as 20 MHz, and the step frequency is 400 kHz to ensure a sufficient number of independent frequencies; that is, $N = 51$ sets of maximums are used to assess the parameters in the empirical distribution.

As for MS, these 48 stirring positions may be not all independent, and consequently one can reshape the array of size 48 into $4 \times 12$ stirrer positions, then $N = 12$ sets of maximums are extracted. It should be noted here that the estimate of parameter $k$ is barely affected despite the existence of correlated samples in these maximums. It can be easily explained that the estimate of $k$ is mainly contributed by resonances. For lower frequencies, there are almost no samples correlated with the extracted 12 sets of maximums in case of the sparse mode, thus will not affect the sign of $k$. For higher frequencies, whether the maximums relate to the same resonance or not, it will also not affect the estimation of $k$. When compared with FS, however, the parameter $s$ (being the variable related to the standard deviation of the maximums) estimated by MS is larger (for the overall frequencies) due to the insufficient number of independent samples.

To estimate the parameters $k$, $s$, and $m$ in (2), L-Ms method is used for maximums extracted by FS and MS configurations, respectively, and the corresponding results are shown in Fig.3. As discussed in this section, the incorrect estimation of parameter $k$ is mainly concentrated in the
undermoded frequencies, since no mode can be excited within the BW for FS, or the stirring procedure for MS. Moreover, fewer numbers of independent maximums also confirm that the MS configuration has a greater uncertainty, resulting in a larger s, as shown in Fig.3.

Moreover, fewer numbers of independent maximums also within the BW for FS, or the stirring procedure for MS. As per [18], we can also use the “well stirred condition” to determine the frequency, i.e., the larger estimated by Anderson–Darling goodness-of-fit test (with a hypothesis that the samples follow the Rayleigh distribution) and by the sample correlation. For the sake of brevity, Fig.4 shows the frequency determined by the Anderson–Darling statistics $A^2_n$ (the threshold is 1.341 for Rayleigh [19], [20]) and the first-order autocorrelation coefficient $(r(1)$ (the threshold is 0.28 [18]), and obviously the larger is 3859.3 MHz, validating the conservative estimates of the EMF behavior with GEV distribution.

B. The maximum EMF

Referring to the GEV distribution, we can estimate the maximums by the quantile $x_p = G_{GEV}^{-1}(p), \,(0 < p < 1)$, i.e.,

$$x_p = \begin{cases} \frac{m - s}{k} \left(1 - \left(\ln p \right)^{-k}\right), k \neq 0 \\ \frac{m - s \left(-\ln p \right)}{k} , \quad k = 0 \end{cases}$$

(7)

It is worth noting that we use the maximum $S_{21}$ samples to estimate the GEV parameters ($k$, $s$, and $m$) for FS and MS configurations, thus the quantile $x_p$ in (7) is the estimated maximum of $S_{21}$. To well model the maximum EMF, we should correct with the mismatch coefficients, e.g., $S_{11}$ and $S_{22}$. As discussed in [5], the maximum EMF $E_{\text{max},p}$ can be derived by

$$E_{\text{max},p} = \frac{4\pi}{\lambda} \frac{5\pi}{\eta_{\text{Tx}} \eta_{\text{Rx}}} \left(1 + |S_{11}|^2 \right)^{1/2} \left(1 + |S_{22}|^2 \right)^{1/2} \frac{x_p}{\ln \left(1 - |S_{11}|^2 \right)^{1/2} \left(1 - |S_{22}|^2 \right)^{1/2}} \quad \text{and} \quad P_{\text{in}}$$

(8)

where $\lambda$ is the wavelength, $\eta_{\text{Tx}}$ and $\eta_{\text{Rx}}$ are the antenna efficiency of Tx and Rx, respectively, $P_{\text{in}}$ is the input power of the enclosure.

As for IEC 61000-4-21, the maximum EMF $E_{\text{max}}$ is derived from the extreme value statistics [2], [21], [22], namely

$$E_{\text{max}} = \left( \frac{4\pi}{\lambda} \frac{5\pi}{\eta_{\text{Tx}} \eta_{\text{Rx}}} \left(1 + |S_{11}|^2 \right)^{1/2} \left(1 + |S_{22}|^2 \right)^{1/2} \right) \alpha(N)$$

(9)

where the symbol $\langle \rangle$ denotes the overall average, $\alpha(N)$ is the function of $N$ as defined in [22]. And specifically, $\alpha(N) = 1.95$ for $N = 12$, and $\alpha(N) = 2.36$ for $N = 50$. For the sake of brevity, we can define $\zeta$ as

$$\zeta = E_{\text{max}} \sqrt{\frac{\eta_{\text{Tx}} \eta_{\text{Rx}} \left(1 - |S_{11}|^2 \right) \left(1 - |S_{22}|^2 \right)}{\alpha(N)}}$$

(10)

Considering a desired confidence of 95 %, the confidence interval $[x_{0.025}, x_{0.975}]$ is used to assess the maximums with GEV distribution. As shown in Fig.5., the maximums related to FS and MS configurations show a good agreement for 500 MHz to 6 GHz. It can be concluded that FS configuration is a good solution to evaluate the maximum EMF for the arbitrary electrically large enclosures without a mechanical stirrer.

At frequencies above the 60th resonance, i.e., 1.5 GHz for the case in Fig.2., $\zeta$ relies on the number of independent maximums $N$, while as discussed in [5], the confidence
interval of the maximum EMF is barely affected by N. Therefore, for the arbitrary electrically large enclosures, GEV distribution can be used to assess the maximum EMF with a desired confidence.

![Comparison of maximums estimated by GEV distribution and IEC 61000-4-21. Specifically, \( \xi_{0.025} \) and \( \xi_{0.975} \) are the values related to \( x_{0.025} \) in FS and MS configurations, respectively, and similarly \( \xi_{0.975} \) and \( \xi_{0.975} \) are related to \( x_{0.025} \). The values \( \xi_{EC,12} \) and \( \xi_{EC,50} \) derived by (9) rely on \( N = 12 \) and \( N = 50 \), respectively.](https://doi.org/10.1017/CBO9780511529443.002)

5. CONCLUSION

In this work, we have shown how to model the maximum EMF for arbitrary electrically large enclosures using the GEV distribution and FS configuration. For this purpose, we make a comparison between the parameters in GEV distribution for both FS and MS configurations, and a comparison between the maximum EMF estimated by GEV distribution and IEC 61000-4-21. The results show that FS configuration can be regarded as a good solution to evaluate the maximum EMF for the arbitrary electrically large enclosures without a mechanical stirrer, and GEV distribution can be used to assess the maximum EMF with a desired confidence.

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Received November 11, 2021
Accepted April 28, 2022