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Accurate Solution of Adjustment Models of 3D Control Network

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Abstract: The spatial Three-Dimensional (3D) edge network is one of the typical rank-lossless networks. The current network adjustment usually uses Least Squares (LS) algorithm, which has the complexity of linearization derivation, computational volume and other problems. It is based on high-precision ranging values. This study aims to minimize the sum of the difference between the inverse distance of the control point coordinates and the observation distance, the composition of the non-linear system of equations to build a functional model. Considering the advantages of the intelligent optimization algorithm in the non-linear equation system solving method, such as no demand derivation and simple formula derivation, the Particle Swarm Optimization (PSO) algorithm is introduced and the improved PSO algorithm is constructed; at the same time, the improved Gauss-Newton (G-N) algorithm is studied for the calculation of the 3D control network adjustment function model to solve the problems of computational volume and poor convergence performance of the algorithm with large residuals of the unknown parameters. The results show that the improved PSO algorithm and the improved G-N algorithm can guarantee the accuracy of the solution results. Compared with the traditional PSO algorithm, the improved PSO algorithm has a faster optimization speed. When the residuals of the unknown parameters are too large, the improved G-N algorithm is more stable than the improved PSO algorithm, which not only provides a new way to solve the spatial 3D network, but also provides theoretical support for the establishment of the spatial 3D network.

Keywords: 3D network, non-linear adjustment model, laser tracker, precision distance.

1. Introduction

The modern large-scale equipment manufacturing industry requires more and more precision in processing and assembly, such as large particle accelerators, ocean engineering equipment, etc., which drives the update and progress of precision engineering measurement technology. As an important part of equipment manufacturing, the high-precision Three-Dimensional (3D) control network has higher and higher precision requirements, mainly relying on high-efficiency and high-precision measuring instruments. Due to the lack of one-dimensional and two-dimensional data, precise 3D coordinate measurement technology is mainly used in high-precision and large-scale devices. The observation values of angles and distances of global control points are determined, and a 3D spatial control network is established. In this way, the measurement efficiency is improved and the operation of the instrument is unified [1]. Numerous scientists have studied how to build a high-precision 3D control network in two ways. On the one hand, more accurate observation values can be obtained by continuously improving the performance of measurement instruments, and on the other hand, the goal of improving accuracy by constructing constraints is achieved in the construction of the network [3], but the solution method of the adjustment model is rarely studied. The error equation of the 3D control network belongs to the non-linear equation set. According to the Taylor series linearization, the second-order and higher-order terms are rounded off. It is then solved as a function of the original Least Square (LS) [5]. Currently, optimization methods for non-linear equations can be divided into numerical algorithms based on Newton-like iterations and intelligent optimization algorithms such as Particle Swarm Optimization (PSO) and genetic algorithms [6].

In recent years, domestic and foreign scientists have conducted extensive research on the 3D network solution method for laser trackers [7], Fan Baixing et al. [4] utilized the tracker distance data with additional center of gravity datum constraints, which greatly improved the point accuracy. Zhai [8] compared and analyzed the advantages and disadvantages of different estimation methods for the problem of the 3D measurement network, providing a reference for a reasonable solution to the problem. Yang [9] solved the initial value of the unknown parameters of the 3D measurement network based on the principle of backward rendezvous and used the method of common point conversion. Xu [10] summarized the Newton method and analyzed the Levenberg-Marquardt (L-M) algorithm to obtain the optimal solution of the non-linear error equation.

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All of the above studies are based on the LS algorithm for net leveling solution, which needs to linearize the non-linear function, requires a large number of complex formulae derivations, and is computationally intensive. To address these problems, this paper proposes an improved Gauss-Newton (G-N) algorithm and an improved PSO algorithm for the 3D measurement network adjustment solution, which reduces the amount of complex derivative calculations in the LS solution process. On the one hand, the Newton-type iterative algorithm mainly utilizes the curvature information provided by the Hessian matrix. There will be some problems in solving non-linear equations. First, the computational set is too large to solve the control network. Therefore, the objective function and its first derivative are used to construct a curvature approximation of the objective function. In doing so, the second-order information in the Hessian matrix is considered to reduce the computational complexity. Second, the G-N algorithm ignores the second-order information in the Hessian matrix, so the algorithm is effective only when the residual function is close to a linear function or zero and the approximate Hessian matrix has at least semi-positive definiteness [11], so a damped G-N is formed by adding a linear search strategy to the G-N. Third, when the objective function is complex, it is necessary to ensure that the objective function is second-order. The damped G-N algorithm is modified by Broyden-Fletcher-Goldfarb-Shanno (BFGS) and can solve the problem that the Hessian matrix loses its positive definiteness and the descending direction cannot be calculated. Finally, an improved G-N algorithm is constructed. On the other hand, the PSO algorithm avoids the complicated derivation process of solving the adjustment model. The computational cost is very low. However, this algorithm can easily fall into the local optimal solution. By combining with other features, the Linearly Decreasing Weight (LDW) strategy and the concept of shrinkage factor to adjust the particle velocity and acceleration [8]-[11] an improved PSO algorithm was developed to prevent it from falling into a local optimum. The 3D edge network is taken as an example, the adjustment model of the 3D control network is solved, the feasibility of the two improved methods proposed in this paper is verified, and the derivation of complex functions in the solution process is avoided, which provides a new idea for solving the adjustment model.

The rest of this article is organized as follows. First, we introduce the theory underlying 3D control networks and formulate the objective function. Then, the method for measuring and calculating the edge network is presented. Next, different algorithms are used to solve experimental systems and the original algorithm is compared with the proposed algorithms. Finally, we draw the conclusions from this work.

2. SUBJECT & METHODS

A. Theoretical basis of 3D control networks

The adjustment model calculation is based on the LS principle [11]. The 3D edge measurement network was taken as an example, and the 3D coordinates of the measurement station and the control point in the global coordinate system were calculated using the observation values from the precision distances. The global coordinates of the i^{th} ($i = 1, 2, \dots, m$) station and j^{th} ($j = 1, 2, \dots, n$) control point are represented by (X_i, Y_i, Z_i) and (x_j, y_j, z_j), respectively, and the distance between the two points is expressed by $S_{ij} (1 \le i \le m, 1 \le j \le n)$. We observe n control points using m stations, and a 3D edge measurement network is constructed, as shown in Fig. 1.

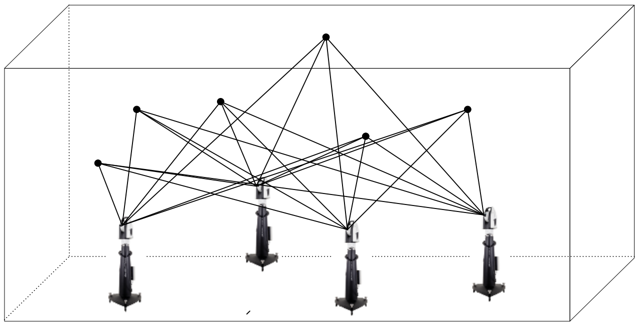


Fig. 1. 3D edge measurement network.

The equation to calculate the observed distance between two points is derived using the global coordinates of the station and the control point.

$$E(S_{ij}{}^2) = (X_i - x_j)^2 + (Y_i - y_j)^2 + (Z_i - z_j)^2 \quad (1)$$

Constructing the objective function

The adjustment model for a non-linear system of equations for a 3D edge measurement network is based on the equation of the observed distance between two points:

$$f_{ij} = (X_i - x_j)^2 + (Y_i - y_j)^2 + (Z_i - z_j)^2 - S_{ij}^{0\,2} = 0$$
$$F(\boldsymbol{X}) = f_{ij} (1 \le i \le m, 1 \le j \le n) \quad (2)$$

where S_{ij}^0 is the measured distance and the unknown parameter

$$\underset{3(m+n)\times 1}{\boldsymbol{X}} = (X_1, Y_1, Z_1, \cdots, X_m, Y_m, Z_m, x_1, y_1, z_1, \cdots, x_n, y_n, z_n)^T$$

is used for $t = 3(m+n)$. When solving the non-linear equations, the global minimum of $\| F(\boldsymbol{X}) \|_2$ (2-norm) is equivalent to the non-linear system of equations $F(\boldsymbol{X}) = 0$ [16]. The global minima of $\| F(\boldsymbol{X}) \|_2$ and $\| F(\boldsymbol{X}) \|_2^2$ are the same; they are defined as

$$\min \| F(\boldsymbol{X}) \|_2^2 = \sum_{i=1, j=1}^{i=n, j=m} \left(\| f_{ij} \|^2 \right) \quad (3)$$

Therefore, (3) is used as the objective function for solving the 3D edge measurement network.

Get the initial value of an unknown parameter

For example, we used a laser tracker to measure the 3D coordinates of the control points according to the principles of spherical coordinate measurement. Laser trackers are widely used in the precision industry and in engineering due to their high measurement accuracy, efficiency and ease of use [3]. For large-scale measurements, single station measurements cannot be used. At present, global control points are usually acquired by multi-station measurements, and the coordinate values of control points in a station coordinate system are determined by spherical coordinate transformations. Table 1 shows the technical parameters of the AT402 laser tracker.

Table 1. Technical parameters of the laser tracker AT402.

Parameter name	Technical parameter	
Measurement space	Max distance	160 m
	Horizontal	± 360°
	Vertical	± 145°
Angular measuremet accuracy	± (15 µm/m + 6 µm/m)	
Preheating time	8 min	
Absolute ranging accuracy	± 10 µm	

Assume that the coordinate values of control point *j* in the coordinate system of the *i^th^* station are (X_{ij}, Y_{ij}, Z_{ij}). The observation values of horizontal angle, zenith angle, and oblique distance from the *i^th^* station to the control point *j* are H_{ij},· V_{ij}, and S_{ij}, as shown in Fig. 2.

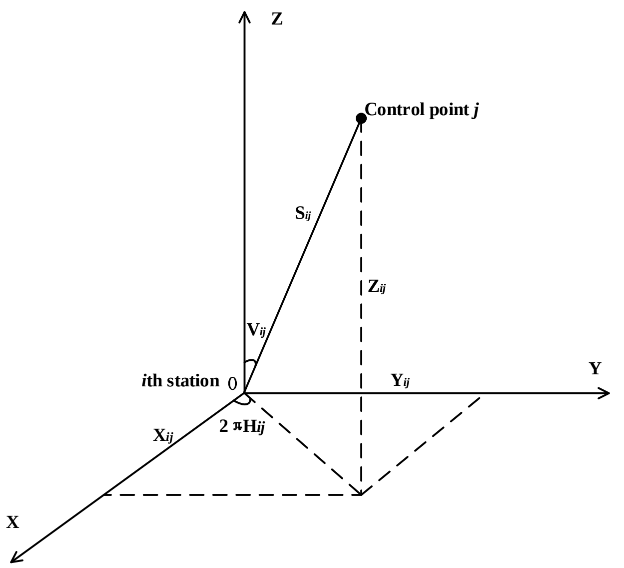


Fig. 2. Principle of spherical coordinate measurement systems.

The coordinate values of the control points in the station coordinate system can then be determined by spherical coordinate transformations.

$$\begin{cases} X_{ij} = S_{ij}\sin\, V_{ij}\cos\left(2\pi - H_{ij}\right) \\ Y_{ij} = S_{ij}\sin\, V_{ij}\sin\left(2\pi - H_{ij}\right) \\ Z_{ij} = S_{ij}\cos\, V_{ij} \end{cases} \tag{4}$$

The first station coordinate system is regarded as the global coordinate system. According to the principle of common point coordinate conversion, at least three or more common points between two adjacent stations are needed to calculate the initial value of each station's position of the global coordinate system.

$$\begin{pmatrix} X_{ij} \\ Y_{ij} \\ Z_{ij} \end{pmatrix} = R_0 \begin{pmatrix} x_j - X_{i0} \\ y_j - Y_{i0} \\ z_j - Z_{i0} \end{pmatrix} \tag{5}$$

where (X_{i0}, Y_{i0}, Z_{i0}) and R_0 are the initial position and posture values of the *i^th^* station of the global coordinate system, respectively. The initial coordinate values

$$\underset{3(m+n)\times 1}{\boldsymbol{X}_0} = (X_{10}, Y_{10}, Z_{10}, \cdots, X_{m0}, Y_{m0}, Z_{m0}, x_{10}, y_{10}, z_{10}, \cdots, x_{n0}, y_{n0}, z_{n0})^T$$

B. Adjustment method

At present, the 3D edge measurement network is mainly a rank-deficient free network adjustment model based on the central datum constraint and is calculated by the LS principle [1]. In this article, the optimization method of non-linear equations is used to solve the objective function of the measurement network. With respect to the adjustment model consisting of non-linear equations, the improved G-N algorithm and the improved PSO algorithm are developed.

The improved G-N algorithm

The Newton-type algorithm uses an iterative approach to calculate the optimal value of the objective function by neglecting third-order and higher-order terms at the initial iteration point using a Taylor series expansion, as follows [17]:

$$f_{ij}(\boldsymbol{X}_k + \boldsymbol{s}_k) \approx f_{ij}(\boldsymbol{X}_k) + \boldsymbol{g}_{ij}^k \boldsymbol{s}_k + \frac{1}{2}\boldsymbol{s}_k{}^T \boldsymbol{G}_k \boldsymbol{s}_k \tag{6}$$

where f_{ij} is the residual function of the distance, $\boldsymbol{X}_k$ is an approximation of an unknown parameter, $\boldsymbol{s}_k = \boldsymbol{X} - \boldsymbol{X}_k$ represents the residuals of an unknown parameter, $\boldsymbol{g}_{ij}^k = \nabla f_{ij}(\boldsymbol{X}_k)$ is the gradient of the distance residual function, and $\boldsymbol{G}_k = \nabla^2 f_{ij}(\boldsymbol{X}_k)$ is the second-order derivative of the distance residual function.

Considering the objective function $\min \parallel F(\boldsymbol{X}) \parallel_2^2 = \sum_{i=1,j=1}\left(\left\|f_{ij}\right\|^2\right)$ of the non-linear system of equations based on the observation distance values to represent an unconstrained minimum problem, $\boldsymbol{X}_k$ is an approximation of an unknown parameter and the gradient of $\parallel F(\boldsymbol{X}) \parallel_2^2$ refers to (7).

$$\boldsymbol{g}_k(\boldsymbol{X}) = \sum_{i=1,j=1}^{i=m,j=n} f_{ij}(\boldsymbol{X}_k)\nabla f_{ij}(\boldsymbol{X}_k) = \boldsymbol{J}(\boldsymbol{X}_k)^T F(\boldsymbol{X}_k) \tag{7}$$

Where $\boldsymbol{J}(\boldsymbol{X}_k)$ is the Jacobian matrix of $F(\boldsymbol{X}_k)$ and $\boldsymbol{G}(\boldsymbol{X}_k) = \boldsymbol{J}(\boldsymbol{X}_k)^T \boldsymbol{J}(\boldsymbol{X}_k) + \boldsymbol{S}(\boldsymbol{X}_k)$ is the Hessian matrix of $\parallel F(\boldsymbol{X}) \parallel_2^2$, where the quadratic form of the objective function $\parallel F(\boldsymbol{X}) \parallel_2^2$ is defined in (8).

$$m_k(\boldsymbol{X}) = F(\boldsymbol{X}_k) + \boldsymbol{g}_k \boldsymbol{s}_k(\boldsymbol{X}) + \frac{1}{2}\boldsymbol{s}_k^T \boldsymbol{G}(\boldsymbol{X}_k)\boldsymbol{s}_k \tag{8}$$

After the right side of (8) is minimum and in order to make its first derivative zero, the Newton iteration formula can be calculated, as referred to in (9).

$$\begin{cases} -\boldsymbol{G}_k^{-1}\boldsymbol{g}_k = \boldsymbol{s}_k \\ \boldsymbol{X}_{k+1} = \boldsymbol{X}_k - \boldsymbol{G}_k^{-1}\boldsymbol{g}_k \end{cases} \tag{9}$$

where X_k and X_{k+1} are the unknown parameter values of the k^{th} and $(k+1)^{th}$ iterations, respectively, α_k and d_k are the step factor and the descending direction of the k^{th} iteration. For the unconstrained optimization problems, the initial values of the unknown parameters must first be provided. The step factor should be calculated according to the minimum problem to determine the direction of descent. Consequently, $\boldsymbol{X}_{k+1} = \boldsymbol{X}_k + \alpha_k d_k$ is obtained. It can be seen that by combining (9) and (7), the step factor of the k^{th} iteration is 1 and the search direction is $-\boldsymbol{G}_k^{-1}\boldsymbol{g}_k$. The G-N method discards the second-order entry $\boldsymbol{S}(\boldsymbol{X})$ in the Hessian matrix $\boldsymbol{G}(\boldsymbol{X})$ and yields the G-N iterative method:

$$\boldsymbol{X}_{k+1} = \boldsymbol{X}_k - \left(\boldsymbol{J}(\boldsymbol{X}_k)^T\boldsymbol{J}(\boldsymbol{X}_k)\right)^{-1}\boldsymbol{J}(\boldsymbol{X}_k)F(\boldsymbol{X}_k) = \boldsymbol{X}_k + d_k \tag{10}$$

The initial values in (10) serve as the starting point for the iterative solution using the G-N algorithm.

The G-N method is usually used to solve non-linear LS problems, but it requires the $\boldsymbol{J}(\boldsymbol{X}_k)^T\boldsymbol{J}(\boldsymbol{X}_k)$ matrix to be at least full rank and needs a descent direction for the linear search. To ensure that the search direction of the objective function decreases at each step, a damped G-N method can be created by adding a linear search strategy to the G-N algorithm and improving (10):

$$\boldsymbol{X}_{k+1} = \boldsymbol{X}_k - \alpha_k\left(\boldsymbol{J}(\boldsymbol{X}_k)^T\boldsymbol{J}(\boldsymbol{X}_k)\right)^{-1}\boldsymbol{J}(\boldsymbol{X}_k)F(\boldsymbol{X}_k) = \boldsymbol{X}_k + \alpha_k d_k \tag{11}$$

where α_k is a one-dimensional search factor that generally assumes a value between 0.5 and 1 [6].

Considering that the matrix $\boldsymbol{J}(\boldsymbol{X}_k)^T\boldsymbol{J}(\boldsymbol{X}_k)$ often becomes singular when the adjustment model of the control network is solved, the first iteration direction d_1 is calculated according to the initial value $\boldsymbol{X}_0$ of the unknown parameter, and the positivity of the iteration direction is determined by the eigenvalue of the parameters. If the eigenvalue is positive, the initial value is included in the iterative process of the algorithm. Otherwise, the algorithm applies the trust region strategy and corrects d_1 using the following approach:

$$d_1 = (\boldsymbol{J}(\boldsymbol{X}_0)^T\boldsymbol{J}(\boldsymbol{X}_0) + \mu\boldsymbol{I})^{-1}\boldsymbol{J}(\boldsymbol{X}_0)F(\boldsymbol{X}_0) \tag{12}$$

where $\mu = 1 \times 10^{-5}$ and $\boldsymbol{I}$ is the identity matrix.

Considering that the algorithm can only solve problems where the initial iteration value is close to the true value of the unknown parameters, the algorithm may not converge if the second-order entry $\boldsymbol{S}(\boldsymbol{X})$ in the G-N method is large. To ensure the feasibility of the algorithm in the presence of a large residual, a BFGS-corrected damped G-N algorithm is proposed. The approximate Hessian matrix is constructed and the secant approximation of the second-order information discarded by the G-N method via the second-order entry $\boldsymbol{S}(\boldsymbol{X})$ is constructed such that the Hessian approximate matrix $\boldsymbol{H}_{k+1}$ of the $(k+1)^{th}$ iteration is closest to $\boldsymbol{H}_k$, and the information of the k^{th} iteration is fully guaranteed.

$$\boldsymbol{H}_{k+1} = \left(\boldsymbol{I} - \frac{s_k y_k^T}{s_k^T y_k}\right)\boldsymbol{H}_k\left(\boldsymbol{I} - \frac{y_k s_k^T}{s_k^T y_k}\right) + \frac{s_k s_k^T}{s_k^T y_k} \tag{13}$$

here $\boldsymbol{y}_k = \boldsymbol{g}_{k+1} - \boldsymbol{g}_k$.

The BFGS-corrected damped G-N algorithm is used to solve the 3D edge measurement network adjustment model. The algorithm flow is shown in Fig. 3.

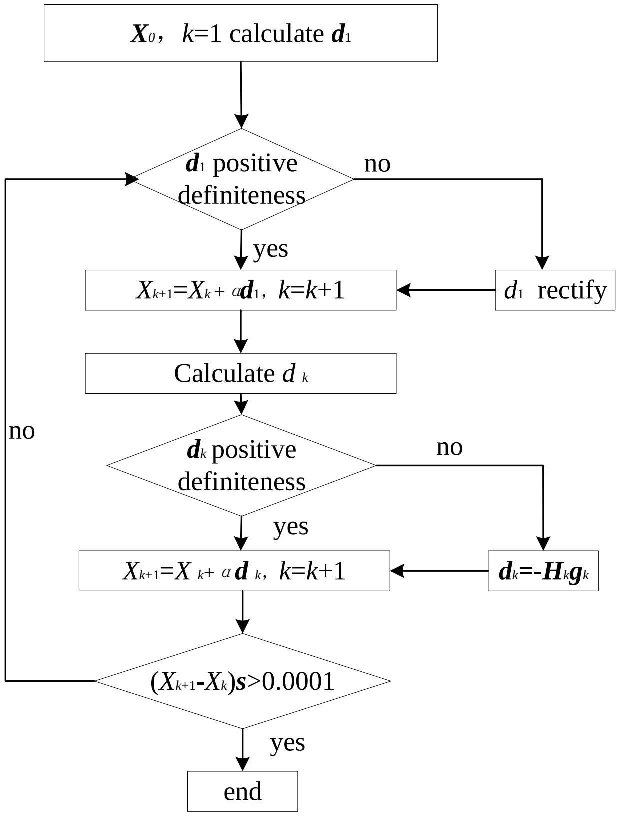


Fig 3. Improved G-N algorithm process.

Based on the principle of spherical coordinate measurements and the method of common point coordinate transformation, the initial values of the unknown parameters are determined, the adjustment is calculated according to the flow of the BFGS-corrected damped G-N algorithm. Set up the iterative calculation formula of the unknown parameters. If d_k is singular, the search direction is not the descending direction, but the trust threshold strategy is used for iterative correction. If $\boldsymbol{H}_k$ is singular, the convergence of the algorithm cannot be guaranteed for large residuals. A secant approximation of the information items of $\boldsymbol{H}_k$ is constructed and the iterative calculation is performed. The 3D global coordinates of the station and the control point are then determined as (X_i, Y_i, Z_i) and (x_j, y_j, z_j), respectively.

The improved SO

The PSO algorithm has its origins in the study of bird predatory behavior. The basic idea of the PSO algorithm is to achieve the optimal solution of the equation through cooperation and information sharing among the individuals in the group [12]. A group of massless random particles is designed by initialization to simulate the birds in the flock, and the optimal solution is obtained by iterative calculation. The particles have only two properties: velocity and position. The velocity indicates the speed of movement, and the position indicates the direction of movement [13]. Each particle independently searches for the optimal solution in the search space and records it as the current individual extreme value. By sharing information with other particles in the entire particle swarm, the optimal individual extreme value is determined as the current global optimal solution of the entire particle swarm, and all particles in the particle swarm update their own velocity and position according to the current individual extreme value and the current global optimal solution shared by the entire particle swarm [13].

There are 3 (m+n) unknown parameters in the space 3D edge measurement network. Suppose that in a large search space with 3 (m+n) dimensions, a community is composed of n particles, where the i^{th} particle is expressed as a 3 (m+n)-dimensional vector:

$$\boldsymbol{x}_i = \left(x_{i1}, x_{i2}, \ldots, x_{i3(m+n)}\right), \quad i = 1,2, \ldots, N.$$

The velocity of the i^{th} particle is expressed as 0, and the optimal solution of the individual and the optimal solution of the entire population are maintained simultaneously. The position and velocity of the ith particle are updated according to (14) and (15):

$$\boldsymbol{x}_{id+1} = \boldsymbol{x}_{id} + \boldsymbol{v}_{id} \tag{14}$$

$$\boldsymbol{v}_{id} = \omega \times \boldsymbol{v}_{id-1} + c_1 r_1 (\boldsymbol{p}_{id} - \boldsymbol{x}_{id}) + c_1 r_2 (\boldsymbol{p}_{gd} - \boldsymbol{x}_{id}) \tag{15}$$

where $\boldsymbol{p}_{id}$ is the individual known optimal solution, $\boldsymbol{p}_{gd}$ is the global known optimal solution, c_1, c_2 are the learning factors, ω is the inertia weight, r_1, r_2 are the random numbers within, $\boldsymbol{x}_{id}$ and $\boldsymbol{v}_{id}$ are the position and velocity of the d^{th} optimization, respectively. The global search ability is strong when the ω value is large, and the local search ability is strong when it is small [14]. Therefore, the strategy of LDW is considered:

$$\omega(t) = (\omega_{ini} - \omega_{\text{end}})(G_k - k)/G_k + \omega_{\text{end}} \tag{16}$$

where k stands for the current iteration times and G_k for the maximum iteration times, ω_{ini} and ω_{end} are the minimum inertia coefficient and the maximum inertia coefficient defined in advance, and are generally 0.4 and 0.9. Optimizing the velocity using LDW results in an improved PSO algorithm that balances the global search ability and local search ability of the algorithm. Accordingly, a larger positive ω value is specified in the initial stage of the algorithm, with ω gradually decreasing linearly as the search progresses. In the later stage of the search, the smaller ω value can ensure that the particles search carefully near the extreme point, so that the algorithm can ensure a greater probability of convergence to the global optimal solution.

To solve the premature problem of the PSO algorithm [15], the concept of shrinkage factor is considered. By adjusting the experience value c_1 of its own particles and the experience value c_2 of the other particles, i.e. the acceleration constant of the algorithm, the step size of the particles flying to the local and global optimal positions is controlled simultaneously, so that the problem of the particles staying too long near the local extreme value is avoided. The shrinkage factor is:

$$\zeta = \frac{2}{\left|h - n - \sqrt{c^2 - 4c}\right|} \tag{17}$$

where $c = c_1 + c_2$. Accordingly, the formula for the velocity is:

$$v_{id} = \zeta \times \omega \times v_{id-1} + c_1 r_1 (p_{id} - x_{id}) + c_1 r_2 (p_{gd} - x_{id}) \tag{18}$$

By optimizing the inertia weight ω and the learning factors c_1, c_2 in the PSO algorithm, the algorithm can reasonably control its search time in local extremes, and at the same time converge faster to the global optimal solution.

With the initial values $\boldsymbol{X}_0$ of the unknown parameters, (3) is solved according to the improved PSO algorithm and the adjustment calculation of spatial 3D edge network is performed. The process is as follows:

1. For the constructed objective function, it is assumed that the population size is N = 40, and the unknown range is set. According to the initial coordinate values $\boldsymbol{X}_0$ of all control points in the global coordinate system, an initial set of random velocities is set, which are generally 0.1-0.2 times their positions. The learning factor value c_1, c_2 is 2.05 and the number of iterations is 2000. Five populations are assumed to have evolved simultaneously.
2. The objective function corresponding to each solution has been calculated according to (3), the current individual extreme value and the global optimal solution of each random solution have been found, the position and velocity of the random solution can be updated according to (14) and (15), and the position and velocity of crossing the boundary can be adjusted.
3. The stochastic solution of the objective function value has been evaluated. Then the historical and global optimal position of the particles can be updated.
4. If the minimum condition is satisfied, the global optimal result is output and the program is terminated. Otherwise, steps (2) and (3) are repeated.
5. The improved PSO algorithm described above solves the spatial 3D edge measurement network.

3. RESULTS

As shown in Fig. 4, 12 control points and 4 stations were set up within a 15×10×3 m space. The temperature is relatively stable. All control points were observed at each station. The measurements are performed with a Leica AT402 laser tracker [18]-[19]. Before the experiment, the instrument was checked to ensure that it meets the nominal accuracy requirements. Each station corresponds to 12 distances from the stations to the point, 4 stations have 48 distances.

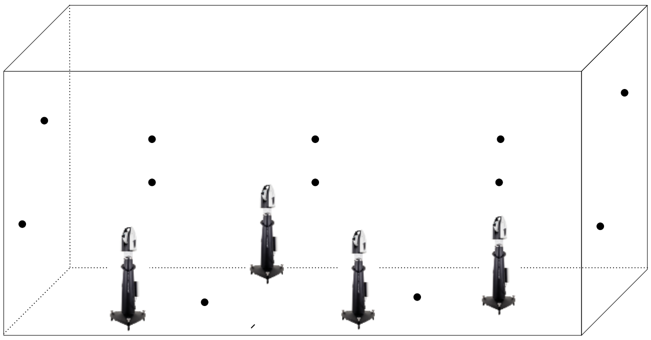


Fig. 4. Layout of the control points and stations.

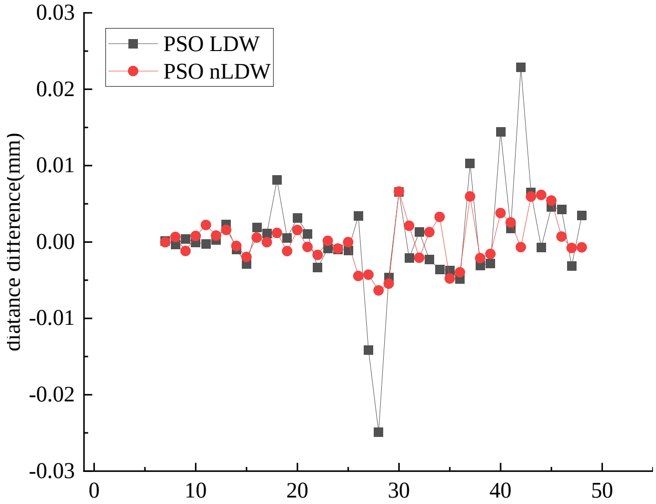
A. *Experiment 1*

The initial 3D coordinates of the stations and control points are obtained by spherical coordinate transformations. The global 3D coordinates of the stations and control points were determined by the rank deficient free network adjustment method with additional barycentric reference constraints [5], the improved PSO method and the improved damped G-N method. The laser tracker has a high precision ranging capability, and the instrument was verified to meet nominal accuracy standards prior to the experiment. The laser tracker's ranging value can be used as a reference value to evaluate the adjustment of the network. In addition to the 3D coordinates of the station and the control points in the global coordinate system, the distance between the station and the control points was calculated according to (18), and then the distance difference ΔD_{ij} and the average value were calculated.

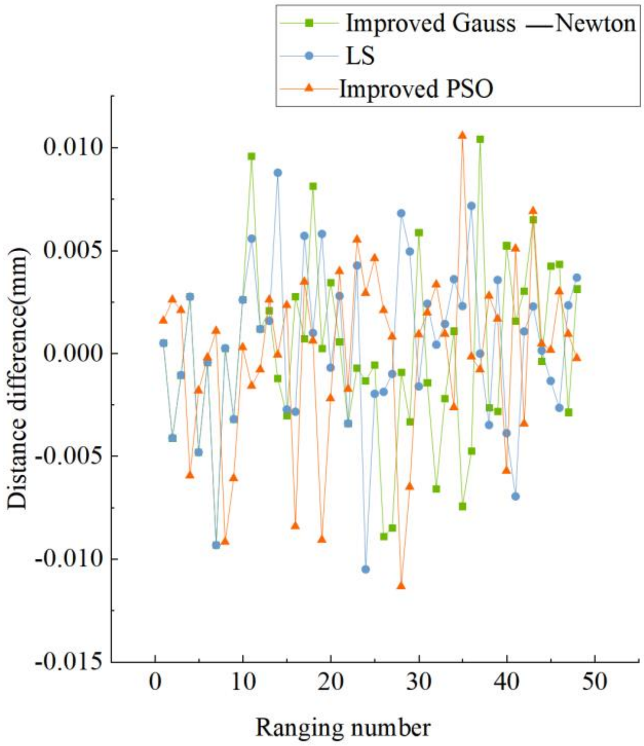
$$\begin{cases} (X_i - x_j)^2 + (Y_i - y_j)^2 + (Z_i - z_j)^2 = S_{ij}{}^2 \\ \Delta D_{ij} = S_{ij} - S^0{}_{ij} \\ \Delta D = \sum_{i=1,j=1}^{i=m,j=n} \Delta D_{ij} \end{cases} \tag{18}$$

The calculation results are shown in Fig. 5, where the rank deficient free network adjustment is abbreviated as LS.

As shown in Fig. 5, the LDW is better than the non-Linearly Decreasing Dynamic Weight (nLDW). The adjustment model of the 3D edge network is solved by the LS, the improved PSO algorithm and the improved G-N method. It is noteworthy that all three algorithms yielded the same 48-segment distances. The maximum distance differences were 0.009, 0.01, and -0.011 mm, and the minimum differences were -0.011, -0.009, and -0.011 mm. The average distance differences for the respective algorithms were -0.0003, -0.0006, and 0 mm. These results confirm the feasibility of the two methods proposed in this work to solve the adjustment model for the 3D edge network.



(a) LDW and nLDW of PSO.

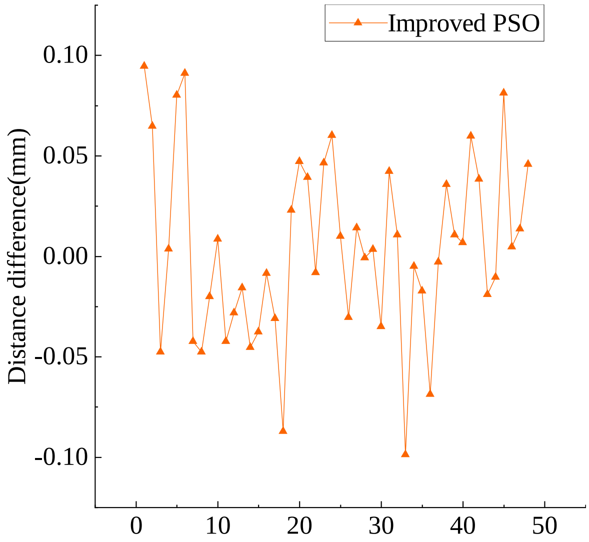


(b) Different algorithms.

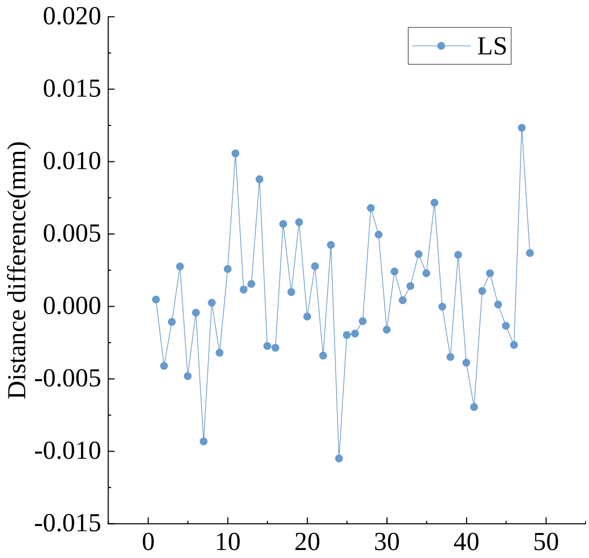
Fig. 5. Calculation results using different algorithms.

B. *Experiment 2*

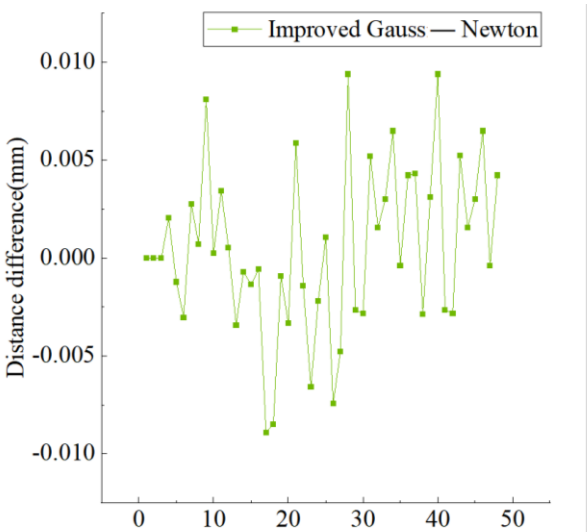
By comparing the initial values of the unknown parameters with the results from the first experiment, we tested the initial values of the unknown parameters with large residuals. The LS [5], the improved PSO algorithm and the improved G-N method are used to calculate the 3D coordinates of the stations and the control points in the global coordinate system. Referring to (14), the difference ΔD_{ij} and the average difference ΔD are calculated. As shown in Fig. 6, the calculation results show the difference between the calculated values and the actual distances of the 48-segment distances.



(a) Improved PSO-Ranging number.



(b) LS-Ranging number.



(c) Improved G-N Ranging number.

Fig. 6. ΔD_{ij}calculated using different algorithms.

The LS [5], the improved PSO algorithm and the improved G-N method required two, four, and two iterations, respectively, for the calculations. When these methods are applied to solve the 3D edge measurement network adjustment model, the effect of the distance difference is as shown in Fig. 6. The maximum values of the distance difference results for the three respective solution methods were 0.011, 0.09, and 0.009 mm; the minimum values were -0.009, -0.098, and -0.011 mm; and the mean values were 0.0007, -0.003, and -0.0004 mm. It is evident that the improved G-N solution method and LS perform significantly better than the improved PSO method. In particular, the improved G-N algorithm showed greater robustness than the other algorithms. This shows that the improved G-N algorithm is able to effectively solve initial unknown parameter values with relatively large residuals.

4. Conclusion

In this article, different solutions for the adjustment model of the 3D control network are investigated. In view of the problems of the existing adjustment methods, such as the large computational volume, the large number of complex function derivation calculations, etc., a non-linear mathematical model is established, the PSO and G-N algorithms are introduced and improved, and the mathematical model is solved. To overcome the limitations of the G-N and PSO algorithms, the BFGS-corrected damped G-N algorithm and the improved PSO algorithm were proposed. Using the 3D edge measuring network as an example, the non-linear equations are constructed based on the distance observations. In addition, the adjustment solution objective function is formed. The analysis of the experimental data proves the effectiveness of the algorithm in solving the adjustment model. In particular, when dealing with large initial residuals, the proposed BFGS-corrected damped G-N algorithm outperformed the LS and improved PSO algorithms in terms of robustness. Therefore, this method provides a new way to solve the adjustment model of the 3D control network. This is of great practical significance for improving the accuracy of the control network solution and performing high-precision industrial and engineering measurements.

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