An Iterative Method for Solving the Inverse Problem in Electrocardiography in Normal and Fibrillation Conditions: A Simulation Study

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Abstract. *Electrocardiographic Imaging (ECGI) is a new imaging technique that noninvasively images cardiac electrical activity on the heart surface. In ECGI, a multi-electrode vest is used to record the body surface potential maps (BSPMs). Then, using geometrical information from CT-scans and a mathematical algorithm we construct the electrical potentials on the heart surface. The reconstruction of cardiac activity from BSPMs is an ill-posed problem. Some algorithms work well in sinus rhythm, but in arrhythmic conditions the reconstruction of the heart potential becomes worse. In this work we present an iterative mathematical approach based on domain decomposition methods and test it on synthetically generated data for normal and fibrillating heart conditions using anatomical 43 years old women geometry.*

Keywords: Electrocardiographic imaging, inverse problem, domain decomposition, bidomain model, ECG modeling

1. Introduction

The inverse problem in cardiac electrophysiology also known as electrocardiographic imaging (ECGI) is a new and a powerful diagnostic technique. It allows the reconstruction of the electrical potential on the heart surface from electrical potentials measured on the body surface. The computation of the electrical potential on the heart surface giving data on the body surface is known to be ill posed in the sense that a small variation of the body surface potential could highly modify the solution on the heart surface. In the mathematical community it is known by data completion as the Cauchy problem. Since 1923, Hadamard [1] has given an example illustrating the ill posedness of this problem. In the electrocardiographic community, different methods of regularization have been used and compared in order to solve the problem. These methods include Thikhonov regularization with L-curve [2], Composite Residual and Smoothing Operator (CRESO) [3], truncated SVD regularization and other regularization techniques (see [4] and the references there). In most of the papers in the literature the used formulation of the inverse problem is based on a transfer matrix that maps the electrical potential on the heart onto the body surface. The matrix is computed using the Green formula if the torso is supposed isotropic and homogenous or by the boundary elements method if the torso is supposed to be isotropic and piecewise homogeneous or by the finite element method in a more general situation in cases where the torso can be supposed anisotropic and/or inhomogeneous.

In this work we go back to the original formulation of the problem and propose a new mathematical way to solve the problem. The method that we present in this paper is based on a domain decomposition technique. It has been recently proposed at the international Conference of Domain Decomposition by Zemzemi [5] and tested on concentric spheres representing the torso. In this work it will be tested on synthetically generated data of a real human torso.

2. Methods

Anatomical Model

The torso geometry was generated from CT images of a 43 years old woman. The DICOM images were segmented using the medical imaging software Osirix. In order to take into account the torso heterogeneity we distinguish three different volume regions (lungs, bones, and the rest). After generating the surfaces we use INRIA meshing Software MMG3D to generate the 3D volume of the computational mesh. In Fig. 1, we show a screenshot of the mesh with the different regions. We assigned isotropic conductivities of 0.389 and 0.2 mS/cm to the lung and bone elements, respectively and 2.16 mS/cm to the rest.

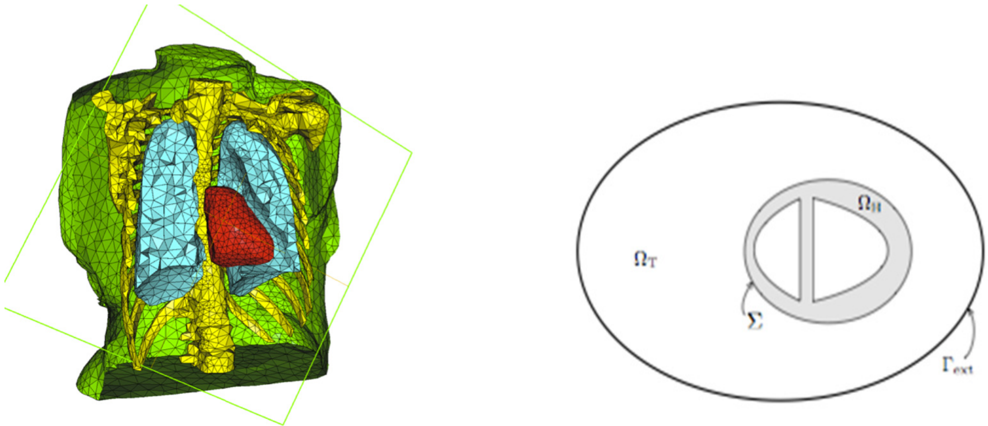

Fig. 1. Left: a cut of the torso computational domain showing the three different regions considered in the torso and the internal and external boundaries of the torso. Right: Two-dimensional schematic representation of the heart and torso domains.

Mathematical Modeling and Numerical Method

Let us suppose that Ω_T denotes the torso domain, Γ_{ext} is the external boundary, Σ is the heart torso interface and u_T is the electrical potential in the torso. The electrical potential on the torso is governed by the diffusion equation. For given electrical potential $\boldsymbol{d}$ on the external boundary, the inverse problem in electrocardiography is to find the electrical potential on the heart surface Σ satisfying both Dirichlet and Neumann boundary conditions on Γ_{ext} as shown in Eq. (1):

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} u_{\mathrm{T}}) = 0, \text{ in } \Omega_{\mathrm{T}}, \\ u_{\mathrm{T}} = d \text{ and } \boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} u_{\mathrm{T}}.\boldsymbol{n} = 0, \text{ on } \Gamma_{\mathrm{ext}}, \\ u_{\mathrm{T}} = ?, \text{ on } \Sigma. \end{cases} \tag{1}$$

Here σ_T denotes the conductivity parameter and depends on the three considered regions. The domain decomposition technique used for solving this inverse problem splits the ill posed problem (1) into two well posed problems:

$$\begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} u = 0, \text{ in } \Omega_{\mathrm{T}}, \\ u = d, \text{ on } \Gamma_{\mathrm{ext}}, \\ \boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} u.\boldsymbol{n} = -\boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} v.\boldsymbol{n}, \text{ on } \Sigma. \end{cases} \qquad \begin{cases} \operatorname{div}(\boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} v) = 0, \text{ in } -\Omega_{\mathrm{T}}, \\ \boldsymbol{\sigma}_{\mathrm{T}}\boldsymbol{\nabla} v.\boldsymbol{n} = 0, \text{ on } -\Gamma_{\mathrm{ext}}, \\ v = u, \text{ on } \Sigma. \end{cases} \tag{2}$$

where $(-\Omega_T)$ and $(-\Gamma_{ext})$ are respectively the symmetrical images of Ω_T and Γ_{ext} through the boundary Σ. The well-posed problem (2) could be mathematically seen as the Poincaré-Steklov formulation of a domain decomposition problem. The only non-classic part in this problem is the fact that we have opposite fluxes at the boundary Σ. Different methods have been presented in the monograph [6] in order to solve domain decomposition problems using iterative procedures. In this paper we use the method presented in [5].

3. Results

In order to generate synthetic data by our ECG simulator based on the bidomain model we simulated two cases: In the first case the heart was stimulated at the apex and the electrical wave propagated from apex to base. We refer to this simulation as a normal case. In the second case we applied S1-S2 protocol in order to produce a re-entry wave. We refer to this case as a fibrillation case. Further information about the forward problem modelling can be found in [7]. We extract the BSP and use it as the input data ***d***. Fig. 2 shows snapshots of the electrical potential distribution in the first and second case for the exact inverse solution. We remark that the wave front is well captured but in the inverse solution it is much smoother than in the exact solution.

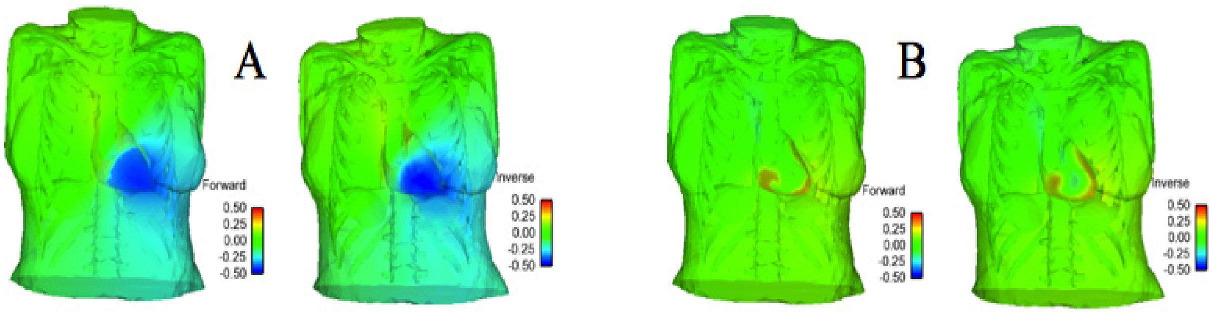

Fig. 2. A: Snapshots of potential distribution in the normal case. B: a snapshot of potential distribution for re-entry case. Forward solution (left) and Inverse solution (right).

In Fig. 3, we show a comparison of the time course of the heart potential between the inverse and the exact solutions at a point located in the right epicardium. The inverse solution is much more accurate in the normal case Fig. 3 (left). In the re-entry case the electrical potential is not accurate in terms of amplitude but looks synchronized with the exact solution. In fact in Fig.4, we show the evolution of the relative error, its mean in time is 0.45. In contrary, the correlation coefficient looks very good (Fig. 4 right) and the inverse solution looks at least 93% accurate in terms of the pattern of activation.

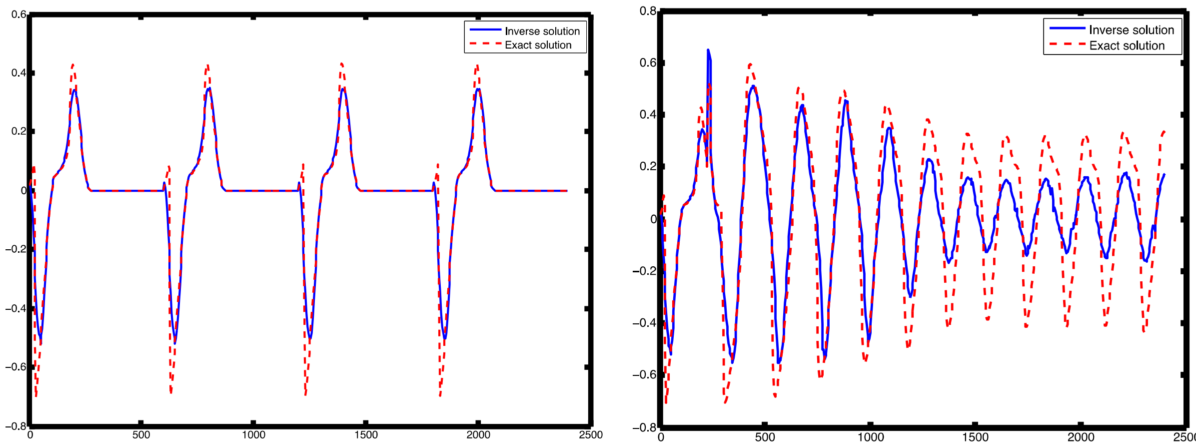

Fig. 3. Comparision of exact (red) and inverse (blue) solutions at a given point on the heart surface for normal (left) and fibrillating (right) heart conditions. X-axis: time (ms) and Y-axis: potential (mV)

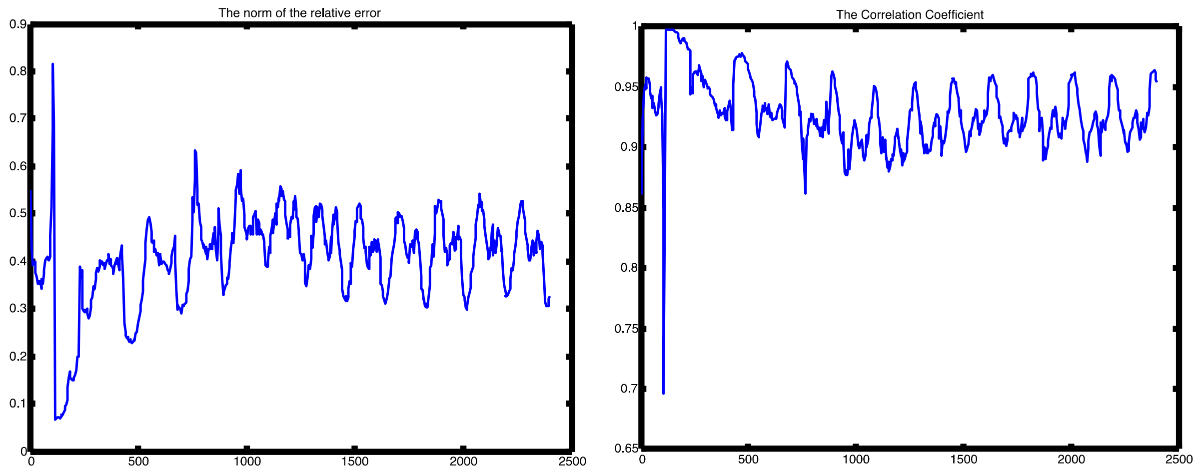

Fig. 4. Time course of the relative error (left) and correlation coefficient (right) computed using the exact and inverse solutions in case of fibrillating heart conditions. X-axis: time (ms) and Y-axis: (dimensionless).

4. Discussion and Conclusions

In this paper, we presented a new approach to solving the inverse problem in electrocardiography. The method is based on a Poincaré-Steklov operator that has solved using a domain decomposition technique. We used a segmented 3D-anatomical model of a 43 years old women torso. We generated synthetic data using the bidomain model for sinus rhythm and fibrillation conditions. In the normal case the reconstruction was very accurate; we have only seen a slight difference in terms of amplitude. In the re-entry wave case, the accuracy is very low in terms of relative error, mainly because the wave front is highly smoothened and the amplitude of the electrical potential is remarkably lower than the electrical potential of the exact solution. In contrary, the mean of the correlation coefficient over the time is 0.93. This gives an important potential for this method to be used in clinical applications.

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