Confidence Interval for the Distance of Two Micro/Nano Structures and Its Applications in Dimensional Metrology

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Abstract. *We propose a method for computing an approximate (Bonferroni type) confidence interval for the distance of two micro- and/or nanostructures considered in dimensional metrology, e.g. grating lines, using data obtained by a length comparator, which allows to determine the coordinates of the edges, the position and the width of the structure from its photometric profile together with the related uncertainty contributions. The implementation is demonstrated using data obtained by the Nanometer Comparator operated at the PTB Braunschweig, Germany.*

Keywords: Length Comparator, Dimensional Metrology, Confidence Intervals

1. Introduction

In many applications the positions and/or the distance of two micro- and/or nano-structures on a surface of a material, as e.g. grating lines, have to be determined together with an appropriate evaluation of the respective uncertainties. Today one-dimensional metrology is typically based on measurements obtained by using highly precise length comparators; as e.g. the Nanometer Comparator implemented at PTB Braunschweig, Germany, see [2, 3].

Here we consider the model for a one-dimensional measurement device where the measurement object is moved and the structure localization sensor (the optical interferometer and the photoelectric sensor unit) is kept in a fixed position. The output of the measurement device relates the photoelectric signal of the sensor, $y(x)$, with the position x of the structure on the measurement object, which is placed on a movable slide of the comparator. For illustration, Fig. 1 depicts the levels of the photoelectric signal $y(x)$ as a function of position x of two 4 μm wide reflective lines (here denoted as the micro-structures), with nominal distance of 30 μm, on a line scale.

In Sections 2 and 3 we will derive and illustrate the construction of the Bonferroni's type confidence interval for the distance of two line structures as an alternative to the standard approach to uncertainty evaluation based on expanded uncertainty. Comparison of those two approaches accompanied with a brief discussion will be presented in Section 4.

2. Subject and Methods

A crucial point in the dimensional metrology of line structures is determination of the coordinates of the structure's edges, the central positions and their respective widths. The coordinate of the left edge is defined as such position x_L that $F = y(x_L)$, i.e. the position of the intersection of selected threshold F and the (increasing part of) photoelectric signal $y(x)$.

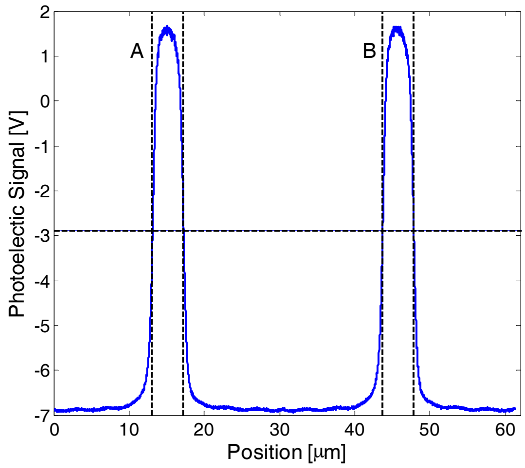

Fig. 1. Photometric profile (signal) of two 4 μm reflex lines A and B with nominal distance of 30 μm. The horizontal dashed line is the threshold F, the vertical dashed lines (from left to right) depicts the positions of the edges ${}_Ax_L$, ${}_Ax_R$, ${}_Bx_L$, and ${}_Bx_R$.

The coordinate of the right edge x_R is such that $F = y(x_R)$, i.e. the position of the intersection of the threshold F and the (decreasing part of) photoelectric signal $y(x)$. The approximate conservative $(1-\alpha) \times 100\%$-confidence interval for the central position and for the edges of the line structures have been considered in [4,5,6]. The suggested method is based on the approximation of the monotonically increasing and/or decreasing sections of the photoelectric signal by linear regression lines.

According to [3-6], the $(1-\alpha) \times 100\%$-confidence interval (CI) for the position of the structure left edge x_L, say $\langle x_{L_1}, x_{L_2} \rangle$, is determined by the following relations:

$$x_{L_1} = \frac{M}{N} - \frac{\sqrt{M^2 - NQ}}{N}, \qquad x_{L_2} = \frac{M}{N} + \frac{\sqrt{M^2 - NQ}}{N}, \tag{1}$$

where

$$N = 1 - \frac{s^2_{\hat{b}_L} t^2_{n_L-2}(1-\alpha/2)}{\hat{b}^2_L}, \; M = \hat{x}_L + \frac{s_{\hat{a}_L\hat{b}_L} t^2_{n_L-2}(1-\alpha/2)}{\hat{b}^2_L}, \; Q = \hat{x}^2_L + \frac{s^2_{\hat{a}_L} t^2_{n_L-2}(1-\alpha/2)}{\hat{b}^2_L}, \tag{2}$$

and

$$\hat{x}_L = \frac{F - \hat{a}_L}{\hat{b}_L} \tag{3}$$

is an estimator of the coordinate of the structure left edge. Here, by $\hat{a}_L$ and $\hat{b}_L$ we denote the estimators of the *left* regression line coefficients (i.e. the intercept and the slope), by $s_{\hat{a}_L}$, $s_{\hat{b}_L}$, and $s_{\hat{a}_L\hat{b}_L}$ we denote the respective standard deviations and the covariance of the estimators, and by $t_{n_L-2}(1-\alpha/2)$ we denote the $(1-\alpha/2)$-quantile (for chosen significance level $\alpha \in (0,1)$) of the Student's t-distribution with $(n_L - 2)$ degrees of freedom, n_L represents the number of data points used for fitting the left regression line. The detailed derivation of the above expressions, as well as the expression for the standard deviation $s_{\hat{x}_L}$ of the estimator $\hat{x}_L$ defined by the equation (3), can be found in [3-6]. The analogical expressions for the *right* edge of the line structure will be denoted by the index R. From that, we directly get the following probability statements for locations x_L and x_R, respectively:

$$\Pr(x_{L_1} \le x_L \le x_{L_2}) = \Pr\left(\frac{x_L}{2} \in \left\langle \frac{x_{L_1}}{2}, \frac{x_{L_2}}{2} \right\rangle\right) = 1-\alpha, \; \Pr\left(\frac{x_R}{2} \in \left\langle \frac{x_{R_1}}{2}, \frac{x_{R_2}}{2} \right\rangle\right) = 1-\alpha. \tag{4}$$

Table 1. Summary results: Evaluation of 12000 photoelectric measurement data pairs.

	A	B		A	B	${}_{AB}d_L$, ${}_{AB}d_R$, ${}_{AB}d$			
x_{L_1}	13.1594	43.7334				${}_{B}x_{L_1} - {}_{A}x_{L_1}$	30.5720		
x_{L_2}	13.1611	43.7351	$\hat{x}_L$	13.1602	43.7342	${}_{B}x_{L_2} - {}_{A}x_{L_2}$	30.5760	${}_{B}\hat{x}_L - {}_{A}\hat{x}_L$	30.5740
95% CI for x_L	0.0017	0.0017	$ks_{\hat{x}_L}$	0.0009	0.0009	95% CI for ${}_{AB}d_L$	0.0039	$ks_{({}_{B}\hat{x}_L - {}_{A}\hat{x}_L)}$	0.0013
x_{R_1}	17.2439	47.8232				${}_{B}x_{R_1} - {}_{A}x_{R_1}$	30.5774		
x_{R_2}	17.2456	47.8248	$\hat{x}_R$	17.2448	47.8240	${}_{B}x_{R_2} - {}_{A}x_{R_2}$	30.5811	${}_{B}\hat{x}_R - {}_{A}\hat{x}_R$	30.5792
95% CI for x_R	0.0017	0.0015	$ks_{\hat{x}_R}$	0.0009	0.0008	95% CI for ${}_{AB}d_R$	0.0037	$ks_{({}_{B}\hat{x}_R - {}_{A}\hat{x}_R)}$	0.0012
x_S									
$(x_{L_1} + x_{R_1})/2$	15.2015	45.7782				$({}_{B}x_{L_1} + {}_{B}x_{R_1} - {}_{A}x_{L_2} - {}_{A}x_{R_2})/2$	30.5745		
$(x_{L_2} + x_{R_2})/2$	15.2035	45.7800	$\hat{x}_S$	15.2025	45.7791	$({}_{B}x_{L_2} + {}_{B}x_{R_2} - {}_{A}x_{L_1} - {}_{A}x_{R_1})/2$	30.5788	${}_{B}\hat{x}_S - {}_{A}\hat{x}_S$	30.5766
95% CI for x_S	0.0020	0.0019	$ks_{\hat{x}_S}$	0.0007	0.0006	95% CI for ${}_{AB}d$	0.0043	$ks_{({}_{B}\hat{x}_S - {}_{A}\hat{x}_S)}$	0.0009
w									
$x_{R_1} - x_{L_2}$	4.0826	4.0879							
$x_{R_2} - x_{L_1}$	4.0865	4.0916	$\hat{w}$	4.0845	4.0898				
95% CI for w	0.0039	0.0021	$ks_{\hat{w}}$	0.0013	0.0012				

We define the centre of the structure as $x_S = (x_L + x_R)/2$ and its width as $w = x_R - x_L$, with their estimators $\hat{x}_S = (\hat{x}_L + \hat{x}_R)/2$ and $\hat{w} = \hat{x}_R - \hat{x}_L$. Their standard deviations are denoted by $s_{\hat{x}_S}$ and $s_{\hat{w}}$, respectively. By using the Bonferroni's inequality and the equation (4) we get

$$\Pr\left(x_S \in \left\langle \frac{x_{L_1} + x_{R_1}}{2}, \frac{x_{L_2} + x_{R_2}}{2} \right\rangle\right) \geq 1 - 2\alpha, \tag{5}$$

and

$$\Pr(w \in \langle x_{R_1} - x_{L_2}, x_{R_2} - x_{L_1} \rangle) \geq 1 - 2\alpha. \tag{6}$$

Now we are interested in the dimensional relations of two line structures. Let us consider the line structures, say A and B, as illustrated in Fig. 1. The related variables, as defined above, will be symbolically denoted by the left-hand sided indices A and B, respectively. The distance of the edges of two line structures A and B is defined as ${}_{AB}d_L = {}_{B}x_L - {}_{A}x_L$, (for left edges), and as ${}_{AB}d_R = {}_{B}x_R - {}_{A}x_R$ (right edges), and finally the distance of two line structures is ${}_{AB}d = {}_{B}x_S - {}_{A}x_S$. Again, by using the Bonferroni's inequality and equations (4-6) we get

$$\Pr\left({}_{AB}d_L \in \langle {}_{B}x_{L_1} - {}_{A}x_{L_2}, {}_{B}x_{L_2} - {}_{A}x_{L_1} \rangle\right) \geq 1 - 2\alpha, \tag{7}$$

$$\Pr({}_{AB}d_R \in \langle {}_{B}x_{R_1} - {}_{A}x_{R_2}, {}_{B}x_{R_2} - {}_{A}x_{R_1} \rangle) \geq 1 - 2\alpha, \tag{8}$$

and finally,

$$\Pr\left({}_{AB}d \in \left\langle \frac{{}_{B}x_{L_1} + {}_{B}x_{R_1} - {}_{A}x_{L_2} - {}_{A}x_{R_2}}{2}, \frac{{}_{B}x_{L_2} + {}_{B}x_{R_2} - {}_{A}x_{L_1} - {}_{A}x_{R_1}}{2} \right\rangle\right) \geq 1 - 4\alpha. \tag{9}$$

3. Results

As an illustration of the suggested confidence intervals for the coordinates of structure‘s edges, their central positions, their respective widths, and their distances we present evaluation of 12000 photoelectric data pairs from two 4 μm wide reflective lines (the line structures), with nominal distance of 30 μm, on a line scale obtained by the Nanometer Comparator implemented at PTB Braunschweig, Germany, see Fig. 1. The summary of the results are presented in Table 1. For comparison, the expanded uncertainties (with the coverage factor $k = 1.96$) are also given.

4. Discussion

In this contribution we have proposed a method for computing an approximate (Bonferroni type) confidence interval for the distance of two micro- and/or nanostructures considered in dimensional metrology, e.g. grating lines, using data obtained by the length comparator. The usually used confidence interval (see e.g. GUM [1]) for the distance ${}_{AB}d = {}_{B}x_S - {}_{A}x_S$, with its length given as $2ks_{({}_{B}\hat{x}_S - {}_{A}\hat{x}_S)}$ (and for the coverage factor $k = 1.96$), is shorter then the suggested Bonferroni type confidence interval, however, as the estimator ${}_{AB}\hat{d} = {}_{B}\hat{x}_S - {}_{A}\hat{x}_S$ is not normally distributed, it does not ensure the stated nominal confidence level (here 95%). On the other hand, the Bonferroni type confidence interval ensures the stated confidence level. The statistical properties of the suggested confidence intervals, especially their coverage probabilities for different experimental setups, are subjects for further investigations.

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