The Method to Evaluation of Calibration for Pressure Transmitter

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Abstract. *The evaluation of calibration for pressure transmitter is based on determination of calibration function in the form of polynomial p degree. The paper describes procedure for evaluation of calibration for pressure transmitter based on finding the polynomial coefficients of the calibration curve by least squares method. This method allows to include uncertainty of a measurement standard pressure to the evaluation of uncertainty by linearization of the model development in Taylor series and neglecting the higher terms. The proposed method is based on conditions of calibration in the Slovak Institute of Metrology. It is compared with methods used in practice not considering uncertainty (standard and influence quantities) for estimation of parameters of the calibration curve.*

Keywords: Calibration, Measurement Uncertainty, Measurement Model

1. Introduction

Calibration of gauges is one of the fundamental tasks in metrological practice. The result of calibration is the assignment of data values of calibrated gauge to the values of measurement standard with the appropriate expanded uncertainty. If the transmitter has a continuous scale then the calibration curve must be found and in most cases it is in the polynomial form. The pressure transmitter is an instrument for pressure measurement (vacuum, pressure, absolute pressure, pressure difference) and the output signal can be in numerical form (numeric display) or in the form of output signal (current, voltage). In practice uncertainties from calibration are not considered for estimation of parameters. They are considered in determining of uncertainty of the estimated parameters of the calibration curve. Further in this paper is presented a procedure for evaluation of pressure transmitter calibration by application of the model for calibration of continuous scale. Assuming that the piston gauge is as a measurement standard then the output signal from the pressure transmitter will be measured by multimeter.

2. Subject and Methods

We consider a theoretical model of calibration in linear form

$$\begin{aligned} p_{\mathrm{E}_1} &= -a - (b-1)p_{\mathrm{sk}_1} - cp_{\mathrm{sk}_1}^2 \\ p_{\mathrm{E}_2} &= -a - (b-1)p_{\mathrm{sk}_2} - cp_{\mathrm{sk}_2}^2 \\ \vdots \quad & \quad \vdots \qquad\qquad \vdots \\ p_{\mathrm{E}_n} &= -a - (b-1)p_{\mathrm{sk}_n} - cp_{\mathrm{sk}_n}^2 \end{aligned} \quad (1) \qquad \text{i.e.} \qquad \begin{aligned} \delta p_{\mathrm{sk}_1} &= a + bp_{\mathrm{sk}_1} + cp_{\mathrm{sk}_1}^2 \\ \delta p_{\mathrm{sk}_2} &= a + bp_{\mathrm{sk}_2} + cp_{\mathrm{sk}_2}^2 \\ \vdots \quad & \quad \vdots \qquad\qquad \vdots \\ \delta p_{\mathrm{sk}_n} &= a + bp_{\mathrm{sk}_n} + cp_{\mathrm{sk}_n}^2 \end{aligned} \quad (2)$$

where

$\delta p_{\mathrm{sk}i} = p_{\mathrm{sk}i} - p_{\mathrm{E}i}$ is an error of calibrated pressure transmitter
$p_{\mathrm{E}i}$ – is a value of measurement standard (piston gauge),
$p_{\mathrm{sk}i}$ – is a value of calibrated pressure transmitter,
a, b, c – are unknown parameters of the calibration function. They are estimated by calibration.

The output signal value of calibrated pressure transmitter measured by multimeter is converted to the unit of pressure using equation (3). It is assumed linear relationship between the output signal and the measured pressure.

$$p_{\mathrm{sk}i} = \frac{\Delta p}{\Delta x}(x_i - x_{\mathrm{d}}) \tag{3}$$

where
Δp – is the span of measuring range for pressure transmitter (Pa),
Δx – is the span of the output signal for pressure transmitter (mA or V),
x_i – is the value of the output signal from pressure transmitter in individual calibration points (mA or V),
x_{d} – is the lowest limit of the output signal from pressure transmitter (mA or V).

Model (2) is non-linear, so it must be linearized at the point (a^0, b^0, c^0, p^0_{ski}) by the development in Taylor series and neglecting the higher terms. Then the calibration model for the *i*-th calibration point is [1, 2, and 3]

$$W_i = a + b\, p_{\mathrm{sk}_i} + c\, p^2_{\mathrm{sk}_i} \tag{4}$$

and

$$W_i = \left(p_{\mathrm{sk}_i} + \Delta p_{\mathrm{sk}_i} + \Delta_{\mathrm{multi}}\right) - \left(p_{\mathrm{E}_i} + C_{m\,zi}\Delta m_{zi} + C_{A\,\mathrm{ef}}\Delta A_{\mathrm{ef}}\right) - d_i\left(\Delta p_{\mathrm{sk}_i} + \Delta_{\mathrm{multi}}\right) \tag{5}$$

where
for simplicity $p^0_{\mathrm{sk}i}$ is taken as $p_{\mathrm{sk}i}$ and . $d_i = -(b^\circ + 2c^\circ \overset{\circ}{p}_{\mathrm{sk}i})$,

$a^\circ, b^\circ, c^\circ$ - initial estimates of the unknown parameters of the calibration function,
$\overset{\circ}{p}_{\mathrm{sk}i}$ - initial estimate of the pressure transmitter pressure (measured value),
Δp_{sk_i} - is an error of the pressure value from pressure transmitter,
Δ_{multi} - is an error of the value of multimeter,
Δm_{zi} - is an error of the weights in *i*-th pressure point,
ΔA_{ef} - is an error of the effective piston area (piston gauge),
$C_{m_{zi}}$ - is the sensitivity coefficient of weights,
$C_{A\mathrm{ef}}$ - is the sensitivity coefficient of the effective piston area (piston gauge),

In matrix form

$$\boldsymbol{W} = \boldsymbol{A}\,\boldsymbol{a} \tag{6}$$

Linear stochastic model

$$(\boldsymbol{W}, \boldsymbol{Aa}, \boldsymbol{U_w}) \tag{7}$$

where $\boldsymbol{W}$ is a vector of input random variables with a mean $E(\boldsymbol{W}) = \boldsymbol{Aa}$ and covariance matrix $D(\boldsymbol{W}) = \boldsymbol{U}_{\boldsymbol{w}}$. $\boldsymbol{A}$ is a known matrix, $\boldsymbol{a}$ is a vector of output variables (the vector of unknown parameters).

Vector of input variables has the following form

$$\boldsymbol{W} = \left(\boldsymbol{P}_{\mathrm{sk}} + \Delta\boldsymbol{P}_{\mathrm{sk}} + \boldsymbol{C}_{\boldsymbol{P}_{\mathrm{sk}}}\boldsymbol{\Delta}_{\mathrm{multi}}\right) - \left(\boldsymbol{P}_{\mathrm{E}} + \boldsymbol{C}_{m\,zi}\Delta\boldsymbol{m}_{\mathrm{z}} + \boldsymbol{C}_{A\,\mathrm{ef}}\,\Delta\boldsymbol{A}_{\mathrm{ef}}\right) - \boldsymbol{D}\left(\Delta\boldsymbol{P}_{\mathrm{sk}} + \boldsymbol{\Delta}_{\mathrm{multi}}\right) \quad (8)$$

Estimation of parameters of calibration function and covariance matrix of a vector of parameters estimation are determined (see e.g. [1, 2]) as $\hat{\boldsymbol{a}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_{\boldsymbol{W}}^{-1}\boldsymbol{A}\right)^{-1}\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_{\boldsymbol{W}}^{-1}\boldsymbol{w}$ respectively $\boldsymbol{U}_{\hat{\boldsymbol{a}}} = \left(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{U}_{\boldsymbol{W}}^{-1}\boldsymbol{A}\right)^{-1}$. For initial parameters estimates of the calibration curve, the parameters are taken that are evaluated without considering the uncertainties, i. e. it is taken the method used in practice. As a rule, two steps of calculation are sufficient. The evaluation for $\boldsymbol{U}_{\boldsymbol{w}} = \sigma^2\,\boldsymbol{I}$ was done for the data from the Slovak Institute of Metrology [3] (Tab. 1). $\boldsymbol{I}$ is the identical matrix, i.e., without considering the uncertainties in estimating the model parameters and also for

$$\boldsymbol{U}_{\boldsymbol{w}} = \left(\boldsymbol{U}_{\Delta\boldsymbol{P}_{\mathrm{sk}}} + \boldsymbol{C}_{\boldsymbol{P}_{\mathrm{sk}}}\boldsymbol{C}_{\boldsymbol{P}_{\mathrm{sk}}}^{\mathrm{T}}u_{\boldsymbol{\Delta}_{\mathrm{multi}}}^{2}\right) + \left(\boldsymbol{C}_{m\,zi}\boldsymbol{U}_{\Delta\boldsymbol{m}_{\mathrm{z}}}\boldsymbol{C}_{m_{zi}}^{\mathrm{T}} + \boldsymbol{C}_{A\,\mathrm{ef}}\,\boldsymbol{U}_{\Delta\boldsymbol{A}_{\mathrm{ef}}}\boldsymbol{C}_{A_{\mathrm{ef}}}^{\mathrm{T}}\right) - \boldsymbol{D}\left(\boldsymbol{U}_{\Delta\boldsymbol{P}_{\mathrm{sk}}} + \boldsymbol{C}_{\boldsymbol{P}_{\mathrm{sk}}}\boldsymbol{C}_{\boldsymbol{P}_{\mathrm{sk}}}^{\mathrm{T}}u_{\boldsymbol{\Delta}_{\mathrm{multi}}}^{2}\right)\boldsymbol{D}^{\mathrm{T}} \quad (9)$$

Table1. Measured and calculated values.

p_{E} (MPa)	p_{sk} (MPa)	δp_{sk} (MPa)	$\sigma_{p_{\mathrm{skm}}}$ (MPa)
0,000000000	-0,002565	-0,002565000	0,000256032
2,000531531	2,002185	0,001653469	0,000108942
4,000948101	4,003240	0,002291899	0,000468368
6,001410389	6,005990	0,004579611	0,000640641
8,001808350	8,006450	0,004641650	0,000701502
10,002307802	10,007280	0,004972198	0,000564381
12,002776912	12,006665	0,003888088	0,000892233
14,003286455	14,005570	0,002283545	0,000976082
16,003723784	16,003535	-0,000188784	0,001160887
18,004197462	18,000715	-0,003482462	0,001318801
20,004635311	19,997730	-0,006905311	0,001588885

Uncertainty of parameters estimation of calibration function by method used in practise is evaluated by statistical analysis for $\boldsymbol{U}_{\boldsymbol{w}} = \sigma^2\,\boldsymbol{I}$ and for uncertainties of measurement standard and multimeter. For the proposed evaluation method of calibration for $\boldsymbol{U}_{\boldsymbol{w}}$ in the form (9) is the uncertainty of the calibration curve determined by applying the law of uncertainty propagation on calibration model (1) respectively (2) for the estimated parameters for $\boldsymbol{U}_{\boldsymbol{w}}$ in the form (9). Results for model (2) are shown in Fig. 1. where $\hat{\Delta}_{p_{\mathrm{sk}i\mathrm{prax}}}$ and $\hat{\Delta}_{p_{\mathrm{sk}i\mathrm{m}}}$ are the estimates of error pressure values of pressure transmitter using the methods in practice. $\hat{\Delta}_{p_{\mathrm{sk}i}}$ is the estimate of error pressure value of pressure transmitter using the proposed method.

$U_{\hat{\Delta}_{p_{\mathrm{sk}i}}}$ is the uncertainty estimate of error pressure value of pressure transmitter using the proposed method. $U_{\hat{\Delta}_{p_{\mathrm{sk}i\mathrm{prax}}}}$ is the uncertainty estimate of error pressure value of pressure transmitter using the methods in practice.

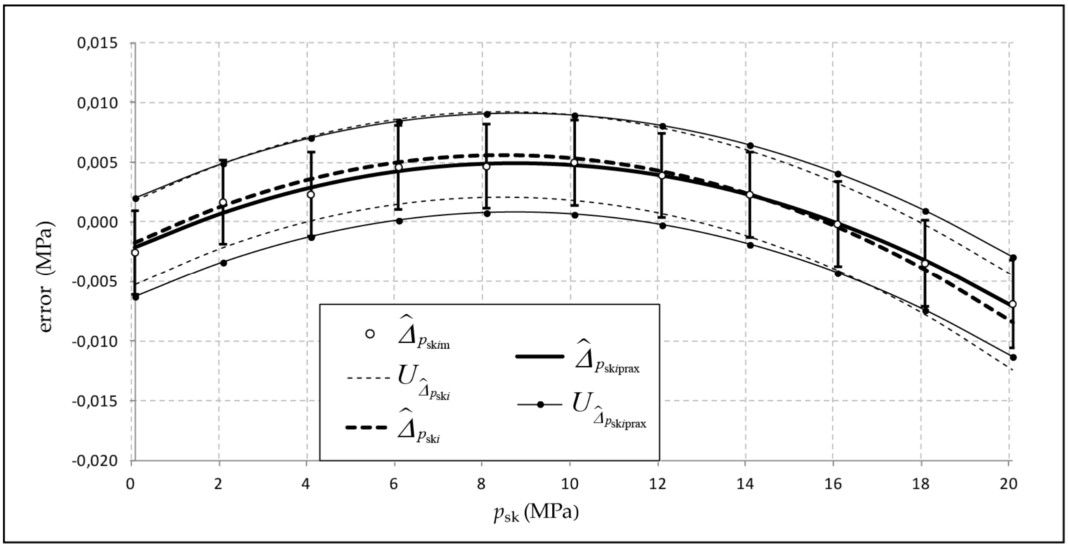

Fig. 1. The graphical presentation of calibration.

3. Discussion and Conclusions

Theory of calibration of continuous scale was applied for the evaluation of the calibration of pressure transmitters. The results of calibration are the estimates of parameters of calibration function and the covariance matrix of the estimated parameters. The results of the evaluation of the calibration using theory for calibration of continuous scale and the results of the evaluation by methods used in practice are slightly different for the conditions considered in this paper. In this case the uncertainties of the estimated values of pressure determined by method B do not affect the estimation of parameters of the calibration function.

Acknowledgements

The authors are grateful for support for this work to the Slovak University of Technology in Bratislava, Faculty of Mechanical engineering, to the Slovak Institute of Metrology and to the Grant Agency APVV - grant No. APVV– 0096-10, VEGA - grant No. 1/0120/12, KEGA – grant No. 005STU - 4/2012.

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