Application of Nakagami Distributions in Ultrasound Contrast Imaging

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Abstract. *Recent developments saw Nakagami and extended Nakagami distributions being used for extracting useful information about ultrasound scenes by matching local histograms from envelope of radiofrequency signals. This information is conveyed by distribution parameters' values. On the other hand, in research literature, new distributions or their extensions are proposed mostly on basis of quality of local histogram match in LMS sense. This measure is not linked in any way to significance of information contained in parameters of these distributions, though. As is shown here, in the context of contrast imaging, the Extended Nakagami distribution significantly improves neither quality of fit, nor information about local area, despite the additional parameter.*

Keywords: Nakagami Distribution, Histogram Fitting, Ultrasound Imaging, Contrast Agents

1. Introduction

Perfusion analysis in ultrasound applications is a modern, dynamical field with many open problems. Ultrasound contrast agents (UCA) in form of various types of microbubbles are used in these applications. The spectrum of echoes returning from UCA contains relatively higher amount of harmonics due to nonlinear behaviour of UCAs. Estimation of perfusion then typically depends on analysis of second harmonic signal, or manipulation of signals such as phase inversion or amplitude modulation, which extract signal content arising from UCA nonlinearities.

An alternative approach to analyse the envelope of the backscattered signal is to model the local statistical properties of the first and second harmonics. One of the general statistical models is Nakagami distribution, which can describe different scattering conditions.

This paper describes an experiment, where ultrasound data are processed with the goal of estimating UCA concentration, particularly the differences caused by generalizing the basic Nakagami distribution to an extended form.

2. Subject and Methods

Nakagami Models

The Nakagami distribution has been originally proposed for modelling multipath-spreading radio signals. As the similarity with ultrasound signal suggests, it is applicable to modelling ultrasound data as well [1]. Its properties, namely ability to exhibit heavy tails, allow the Nakagami distribution to match real data better than e.g. Rayleigh or Gamma distribution. In its basic form, Nakagami distribution is expressed as

$$f(x) = \frac{2m^m x^{2m-1} e^{-\left(\frac{m}{\Omega}x^2\right)}}{\Gamma(m)\Omega^m}, \quad (1)$$

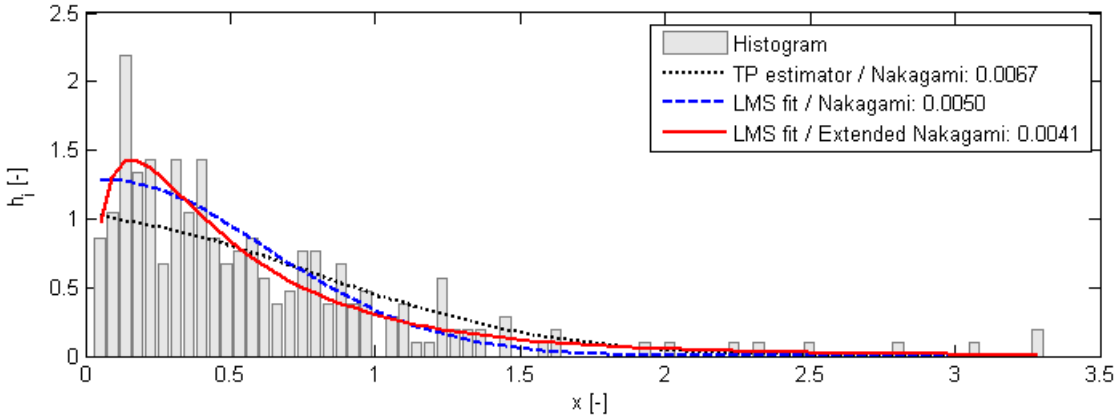

Fig. 1. Typical results of fitting Extended Nakagami and Nakagami distributions to a histogram, along with parameter estimation. Numbers in legend correspond to LMS quality of match.

where x is the independent variable, Γ is the gamma function, m is the shape parameter and Ω the scaling factor. There exist various estimators for the Nakagami distribution [2], among them the Tolparev-Polyakov (TP) estimators:

$$\widetilde{\Omega} = \frac{1}{N}\sum_{i\in ROI} x_i^2 \,; B = \left(\prod_{i\in ROI} x_i\right)^{1/N};\ \widetilde{m} = \frac{1+\sqrt{1+\frac{4}{3}ln\left(\frac{\mu_2}{B}\right)}}{4\,ln\left(\frac{\mu_2}{B}\right)}. \tag{2}$$

To adapt the probability density function (PDF) for heavy tails, common for PDF of ultrasound envelope signals, the variable X describing returning signal can be transformed as $Y = X^{2/s}$. Thus, the extended Nakagami distribution is obtained, with PDF in the form of

$$f(x) = \frac{|s| m^m x^{sm-1} e^{-\left(\frac{m}{\Omega}x^s\right)}}{\Gamma(m)\Omega^m}. \tag{3}$$

The parameter s does not have any direct physical meaning; with its value of 2 the PDF reverts to the basic Nakagami distribution. It can be shown that this generalization of Nakagami distribution is identical to a generalization of Gamma distribution.

Previous research shows that for the (basic) Nakagami distribution, the parameter m (shape parameter) depends linearly on UCA concentration in certain range [3].

Experiment Setup

To evaluate a range of ultrasound contrast agent (UCA) concentrations, an experimental measurement setup was designed. A SonoVue (Bracco International, BR 14) UCA solution contained in a plastic bottle was immersed in a tank filled with water. The initial concentration was 100 mg/ml. During the experiment it was lowered seven times, always halving it, to final concentration of 0.78125 mg/ml. Thus, the range that can be expected in clinical applications was covered.

A conventional ultrasound scanner (GE Vingmed System Five) was used to collect the data. The system was set to base frequency (first harmonic) of 2.4 MHz, mechanical index 0.1 and focus to 7 cm, which corresponded to plastic container position.

Data Processing

Using the previous setup, radiofrequency (RF) data were collected and bandpass-filtered for 1st and 2nd harmonic components. From these datasets, the respective images were

reconstructed. Values of these grayscale images were then normalized by scaling in such way that the maximal value was always converted to 100.

Nine regions of interest were selected within the area of plastic container for further processing. Histograms from image data in these regions (75 bins) were scaled to allow fitting with PDFs, according to Eq. 4.

$$\Delta = \frac{\max(h) - \min(h)}{bins}, \; h' = h \cdot \Delta \tag{4}$$

The histograms were then fitted with Nakagami and Extended Nakagami PDFs, along with estimating (basic) Nakagami parameters by TP estimator. For fitting, least mean square (LMS) minimization has been employed using functions from Matlab Optimization toolbox. A single example of results of this step is shown in Fig. 1. For each of the parameters gathered from all nine regions, median was calculated at each concentration as the final value.

Apart from distribution parameters, we obtained the quality of fit in LMS sense. This allows a direct comparison of the three methods used, after normalization according to Eq. 5.

$$err_{norm} = err_{fit} \cdot \Delta^2 \tag{5}$$

3. Results

As can be seen, the Nakagami PDFs fit the local histograms of obtained data with comparable success. The resulting parameters and LMS error of distribution can be seen in Fig. 2 for 1st harmonic images. Both models give similar results for both harmonics, therefore the following discussion will consider only 1st harmonic images.

Of particular interest is Fig. 3, showing in detail the range described in previous work (1.5625-12.5 mg/l), where the parameter *m* exhibits linear dependence on UCA concentration (R^2=0.97 for estimator and 0.98 for LMS fitting).

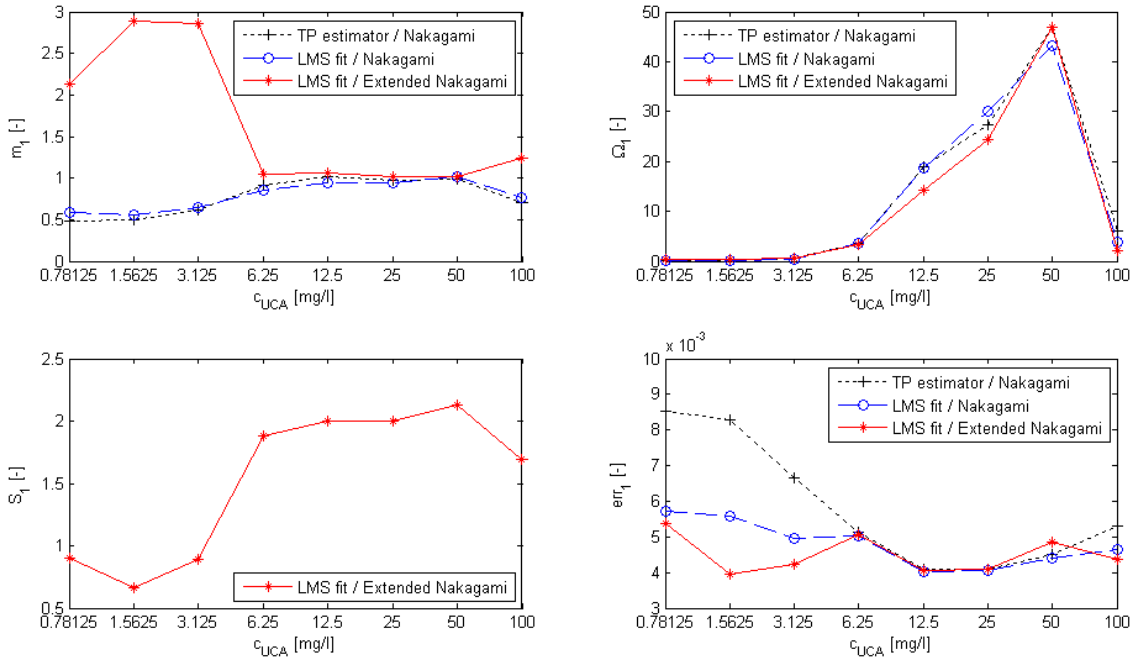

Fig. 2. Parameter values and LMS error of various methods for the first harmonic component. Second harmonic behaves similarly and is omitted for clarity. The values shown are medians calculated from values computed for all nine ROIs. All charts have on x axis the concentration in log scale.

4. Discussion

As can be seen from Fig. 2 and 3, the linear relation between UCA concentration and value of shape parameter *m* is valid for both estimation methods using basic Nakagami distribution. Of note is also that in range 6.25-50 mg/l, the parameter *s* stays very close to its default value of 2, meaning that the Extended Nakagami distribution reverts back to its non-generalized version.

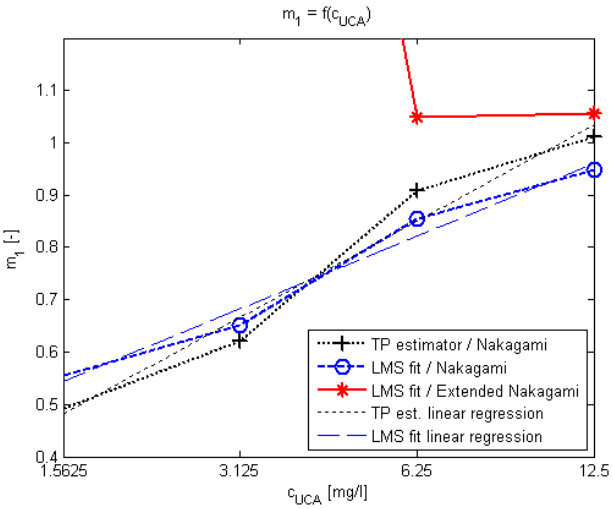

Fig. 3. Detail of parameter m_1 in range where it depends linearly on concentration.

All three methods result in different quality of fit in LMS sense, with LMS fit of Extended Nakagami distribution performing best of the three methods. The improvement is strongest in lower concentrations, with roughly 33% improvement of the LMS-error from fitting basic Nakagami dist. and 53% improvement from TP estimator in best case.

Fig. 3 shows also that the Extended Nakagami distribution does not exhibit the linear dependency of parameter *m* on UCA concentration at all, unlike basic Nakagami distribution.

5. Conclusions

We have shown that the Extended Nakagami PDF leads to lower LMS errors for lower concentration. However, the improvement of fitting quality is not significant (Fig. 2). It should be also mentioned that more parameters (additional parameter *s*) increase the computation complexity, which can lead to instability during fitting procedure. Given the lack of additional parameter's physical interpretation, a phenomenon already criticized [4], the Extended Nakagami distribution does not appear to be useful in contrast imaging.

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