Uncertainty Analysis of Pulse Transient Method for Cylindrical Samples

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Abstract. *We have derived several models for the pulse transient methods which accounted for effect of heat losses from the sample surface as well as the geometrical arrangement of a real experiment. The models were derived for cylindrical as well as cuboid shape of specimens [1], [2]. In the past there was published an uncertainty analysis of the models for cuboid samples. In this paper the same analysis was extended for infinitively long cylindrical specimen. The analysis was developed using the theory of sensitivity coefficients. A theoretical calculation of the model parameters uncertainties and derived analytical formulas are presented. Analysis of the measurement error on the base of sensitivity coefficients show propagation of input uncertainty into calculated parameters. Analysed results shows limitations relating to a range of model validity for of non-stochastic dynamic process. The analysis improves the accuracy of the measurements.*

Keywords: Thermophysical Measurements, Dynamic Methods, Uncertainty Analysis

1. Introduction

The experimental problems connected with geometry of specimens are sometimes the results of having limited size of the tested material. This could cause some problems in data evaluation, because the ideal model most often assumes infinitively large media. Typically, the shapes of the specimens used for the measurement are of cylindrical or cuboid forms. The limited amount of the specimen causes an additional effect that decreases the accuracy of the measurement. The contributions to uncertainty come from additional effects caused by the differences between the ideal and the real size of the sample. The main effect is caused by the heat losses from the sample surface and it is included in model.

2. The principle of the Pulse transient method and the model for cylindrical samples of infinite length

The Pulse Transient method [3] is a dynamic method for the measurement of thermophysical parameters. The principle is based on the measurement of the temperature response to a heat pulse generated by a planar heat source generated. Temperature response is recorded by the thermocouple placed apart from the heat source (Fig. 1). Heat losses effect included in this model is represented by heat transfer coefficient α from the sample surface to the surrounding. Thus, the planar isotherms on Fig. 1 of the heat front are deformed during the measurement. The data that are measured within the marked white area in Fig. 1. are still able to be evaluated by the ideal model. In the case when the thermal isotherms are deformed by this effect we need to introduce new models. The solution of the heat transfer equation for the initial and boundary conditions shown in Fig. 2 is

$$T(t,x,r=0)=T_0\frac{R}{x}\sum_{\xi}\frac{\beta}{\xi(\xi^2+\beta^2)}\frac{1}{J_0(\xi)}\left[e^{-2uv}\Phi^*(u-v)-e^{2uv}\Phi^*(u+v)\right] \quad (1)$$

where $T_0=\frac{qx}{\lambda}$, $\beta=\frac{R\alpha}{\lambda}$, $u=\frac{x}{2\sqrt{\kappa t}}$, $v=\xi\frac{\sqrt{\kappa t}}{R}$, and T temperature response described by model, x axial space coordinate, r radial space coordinate. In this case the thermocouple is placed on the axis so the value of $r=0$. R is the radius of the sample, q heat flow density, λ thermal conductivity, κ thermal diffusivity, α heat transfer coefficient for sample–ambient interface, Φ^*(u) is the complementary error function, ξ is the root of equation $\beta J_0(\xi)-\xi J_1(\xi)=0$ and J_0 and J_1 are the Bessel functions.

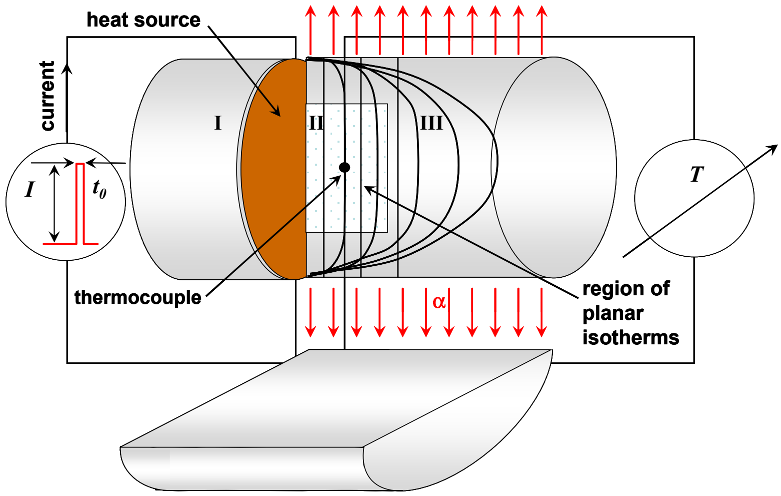

Fig. 1. Wiring diagram and the sample set in a cut. In between first and second part of a sample set a planar heat source is inserted. The thermocouple for the measurement of temperature response to the heat pulse is inserted in between the second and the third part.

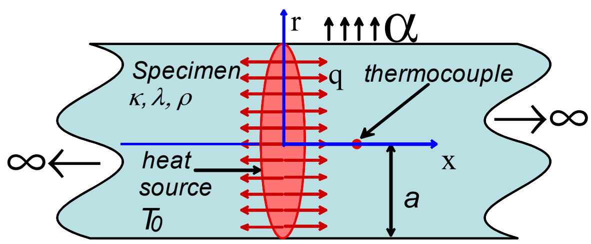

Fig. 2. Initial and boundary conditions for the model assuming two semi-infinite cylindrical specimens and circle shaped heat source in between them.

3. Accuracy estimation results

Generally, uncertainty has different sources and includes errors in the data measurements, parameter estimation procedure and model structures. The uncertainty analysis evaluates how these errors are propagated through the model and calculates their relative importance which is quantified via sensitivity analysis. This type of uncertainty should be supposed as a systematic error of the model. The analysis uses the sensitivity coefficients data derived from the model, along with the underlying data covariance to assess the degree of similarity (linear dependence) between sensitivity coefficients [4] calculated for free parameters. If the sensitivity coefficients are linearly dependent on each other, the parameters could not be estimated unambiguously and thus their uncertainty is high [4]. The general mathematical background for different shape of samples was already published [1], [2]. The final formula for relative uncertainty of a model parameter a_k has the form

$$u_r(a_k)^2 = C_{kT}^2 \frac{u(T)^2}{a_k^2} \tag{3}$$

where u_r is the relative uncertainty, a_k the tested parameter, C_{kt} is the coefficient that represents the projection of the temperature uncertainty to the uncertainty of the tested parameter [4]. The normalized sensitivity coefficients $\beta_{a_i} = a_i \partial f / \partial a_i$ for the discussed models were calculated for the values of thermophysical parameters close to those measured in experiment performed on sandstone [2].

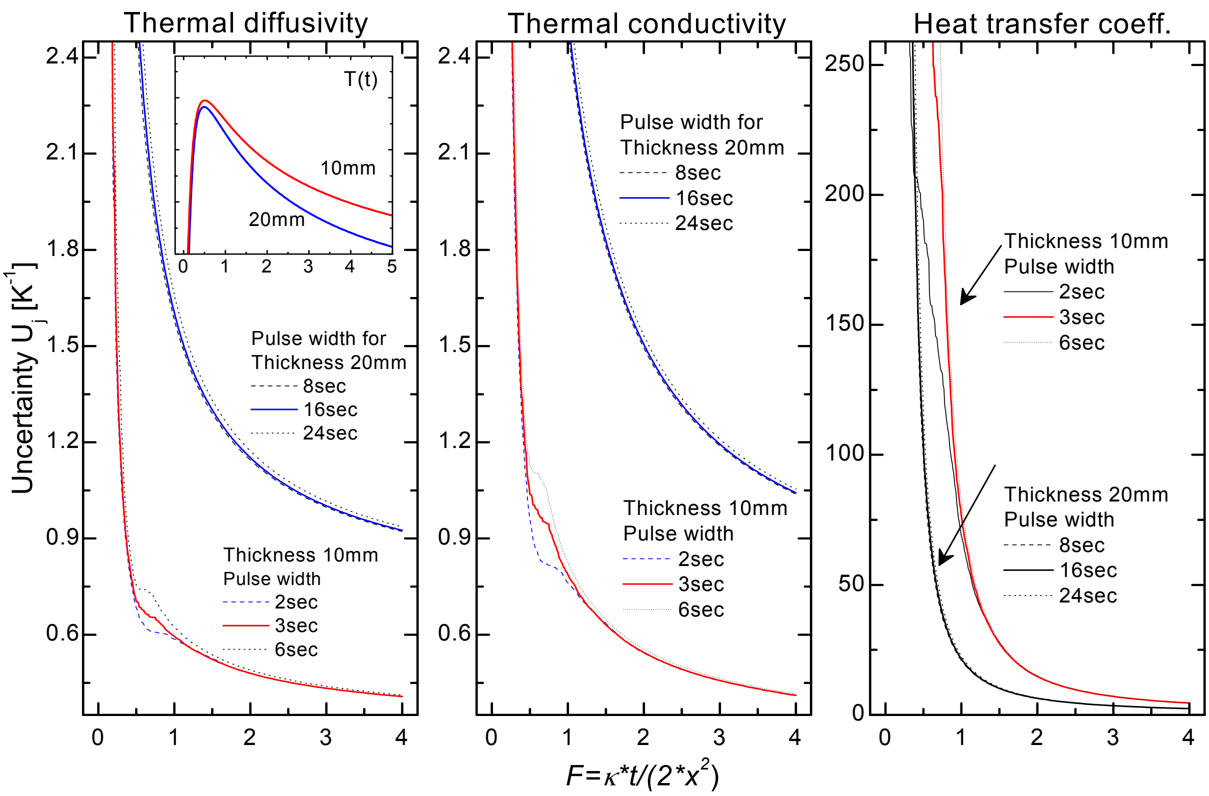

Fig. 3. Uncertainty analysis for sample thickness of 10mm at 2, 4 and 6 seconds of pulse width and thickness 20mm for 8, 16 and 24 seconds of pulse duration. For the illustration of time relation the maximum of temperature response T(t) drawn in blue and red solid lines at 0.5 of the dimensionless time F (Fourier number), that correspond to 42 seconds in real time. Uncertainties in measurement of temperature are propagated through the model and their relative importance is evaluated as uncertainty in K^{-1} for. Relative uncertainty U_j is defined by $U_j = u_j(a_j)/(a_j u(T))$ given in K^{-1} where $u(T)$ is uncertainty of measurement of temperature response.

4. Discussion

The sensitivity coefficients for thermal diffusivity and thermal conductivity are of very similar behaviour like in the case of cuboid samples [2] and have a maximum at the lower times of the temperature response record. The values of the sensitivity coefficient of heat transfer coefficient are increasing in time, so higher sensitivities are expected at longer measurement times. This concludes that for the evaluation of the feasible data from this model we need to perform measurements long enough exceeding times more than 3F. This is confirmed by the uncertainty analysis where the lowest uncertainties are for the dimensionless times evaluated as Fourier number higher than 1.5. For F=3 the uncertainty is 3.55 and for F=4 the uncertainty is 2.4 K^{-1}. The sensitivity analysis in Fig. 3 shows the acceptable error of parameters evaluation for thermal diffusivity and thermal conductivity also for times lower than 1 F. The heat transfer coefficient uncertainty decrease with increasing time of measurement. The uncertainties in Fig. 3 are calculated as uncertainties in respect to input uncertainty of measured temperature. The typical value of temperature uncertainty *u(T)* measured by a thermocouple is about 0.01K. For the evaluation of percentage relative uncertainty we can recalculate uncertainty in the following way $U_{\%} = U_j u(T) * 100\%$.

5. Conclusions

The uncertainty analysis of the pulse transient model for cylindrical samples of infinite length in respect to the heat transfer coefficient was performed and illustrated in the Fig. 3. The accuracy of the results depends on the uncertainty of measured temperature response and in the presence of the effect of the heat losses from the sample free surface depend on the time of the measurement as well as geometry of the specimen. This parameter is affecting the measurement with increasing time of the measurement at larger thicknesses of the sample. The ideal model overestimates values of thermophysical parameters more than ten percent. The heat transfer coefficient is not possible to estimate unambiguously for short times of the measurement because of low sensitivity. The heat transfer coefficient from the sample surface to the surrounding is temperature dependent [4]. The described method of uncertainty analysis is applicable to any kind of the physical model.

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