

## Method for Validation of Antenna Calibration Measurements

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**Abstract.** *Antenna calibration measurement is a specific type of multidimensional measurement with a plenty of influencing factors. In order to validate these measurements, the specialized methodology with a goal of improving the calibration procedures has been developed. The method is based on cluster analysis of existing antenna calibration measurements in a combination with tailored outlier detection method. The paper describes proposed methodology of the validation of individual measurements.*

*Keywords: Antenna Calibration, Measurement Validation, Data Clustering, Outlier Detection*

### 1. Introduction

This paper is focused on the analysis and validation of antenna calibration data. The goal is to examine about 6000 antenna calibration measurements collected in a period of last 14 years. Having access to this unique dataset, various antennas models and calibration methods combinations can be examined.

Our first task was to collect and organize the calibration data and make them ready for further processing. The preliminary analysis of the obtained data has shown that the calibration curves, associated with the same antenna model and calibration method, tend to form natural clusters in praxis. Being able to recognize these groups, it would be possible to determine if a specific antenna diverts from the others of the same model, if calibration data of a specific measurement are valid or not, and more. For the purpose of the data separation a tailored cluster analysis method has been introduced.

### 2. Subject and Methods

Essential part of the data processing was to obtain all the calibration information from distributed heterogeneous archives. Involving software engineering techniques, we were able to find automatically up to 75% of all calibration certificates and from 80% of them we succeeded in extracting also the quantitative calibration data. As a result we have created a database containing thousands of antenna calibrations of different antenna models, performed according to variety of calibration methods.

As proposed in [1], we have chosen the agglomerative hierarchical clustering using the Euclidean distance measure, for the purpose of separation of the antenna factor curves into distinct groups. The crucial property of this clustering approach is that the final number of clusters must be known in advance. When the number of clusters is not explicitly known, one needs to estimate this number empirically from the data. This is, however, a problem on its own for which a number of techniques could be employed. In [1], we stated that the total cluster variance criterion delivers acceptable results. Nevertheless, further experiments on larger dataset have shown limits of this approach. One of the most prominent issues was its inability to detect just a single cluster. Experiments with the real data have shown that the total cluster variance of one cluster is always larger than two smaller clusters. To be able to detect number of clusters in our data correctly, some other criterion had to be considered.

In this paper, we propose another approach how to formulate the clustering criteria based on the expert knowledge. In general, experts recognize false calibrations as those with significantly different shapes with respect to all other curves in the group or if their bias from the others is large. The cases where only small part of the curve differs cannot be considered as invalid because there is possibility that the specific antenna was calibrated in unusual frequency range on special customer request. In our proposed algorithm we attempted to transform these general rules to an autonomous method for determination of number of clusters in the data.

Fig. 1. shows an example set of multiple calibration measurements (curves) obtained for a specific antenna model and calibration method over time. One can see that an outlier in some respect does not necessarily mean an invalid calibration measurement. It can just belong to another subcluster within the given set which can still be considered as valid. This is, however, in a slight contradiction with the typical outlier scenario, where any deviation from the global model is automatically understood as an erroneous measurement. In our case, the nature of the antenna calibration allows for multiple clusters of measurements, although belonging to the same antenna model and calibration method. Thus the elements of one cluster may pose outliers to the other clusters, but still represent valid measurements.

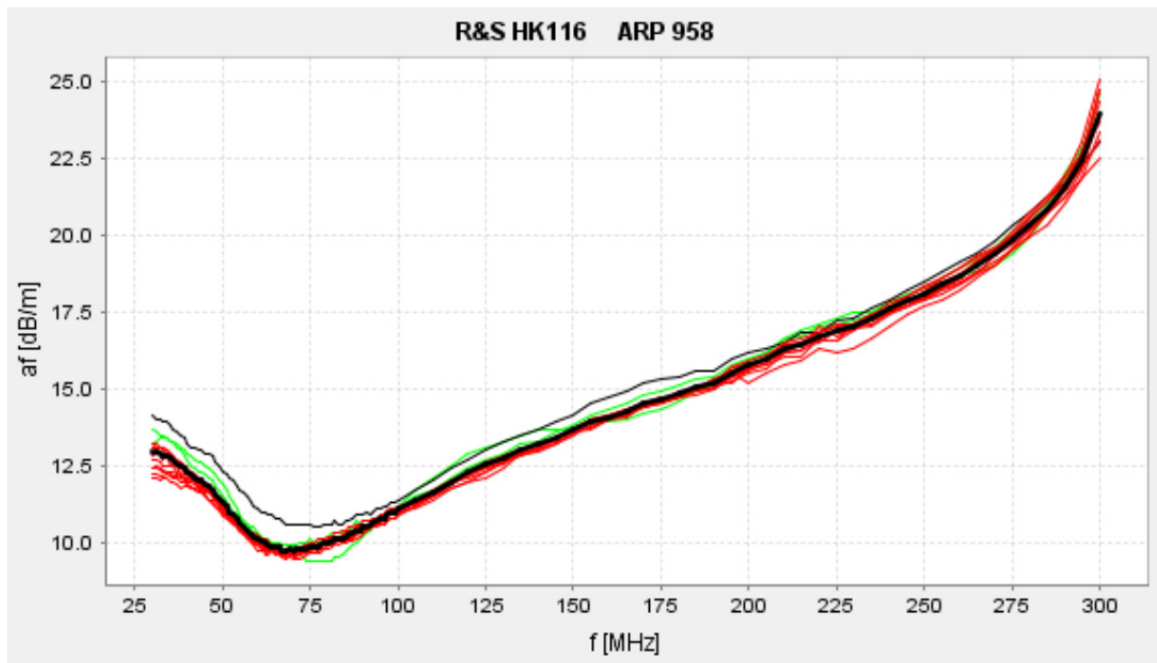


Fig. 1. Example of multiple calibration measurements (curves) obtained for a specific antenna model and calibration method over time. Color groups stand for results of the cluster analysis applied to the data.

The traditional approach to multivariate outlier detection is typically based on the Mahalanobis distance [3]:

$$D_M(x) = \sqrt{(x - \mu)' S^{-1} (x - \mu)}.$$

This measure gives the distance of a vector  $x$  from the data mean  $\mu$ , taking into account correlations between variables reflected by the covariance matrix  $S$ . Assuming that the data follow the multivariate normal distribution, it can be shown that  $D_M^2 = \chi^2(k)$ , where  $\chi^2(k)$  is the Chi-squared distribution with  $k$  degrees of freedom, which is equal to the dimensionality of the data. Given the vector  $x$ , comparing  $D_M^2(x)$  with  $(1 - \alpha)$ -th quantile of  $\chi^2(k)$  provides an easy-to-use tool for assessing whether  $x$  complies with the model at the significance level  $\alpha$  or not. Note that the model is fully defined by the mean vector  $\mu$  and the covariance matrix  $S$ , which must be known either explicitly from an expert knowledge or via estimation from the training data.

However, in the case of antenna calibration measurements, there is always much less calibration curves in each data group than the number of measured frequencies. That causes the covariance

matrix  $S$  of the trained model to be inevitably singular and the inversion  $S^{-1}$  having no exact analytical solution. In order to resolve this issue, we have considered another regularizing assumption that the errors are independent in each measured frequency. This assumption reduces the Mahalanobis distance into the following simpler form:

$$\widehat{D}_M(x) = \sqrt{\sum_{i=1}^k \left( \frac{x_i - \mu_i}{\sigma_i} \right)^2},$$

where  $x_i$  is  $i$ -th element of the calibration curve  $x$ ,  $\mu_i$  is  $i$ -th element of the mean vector  $\mu$  and  $\sigma_i$  is the standard deviation of the data in  $i$ -th measured frequency. Comparing  $\widehat{D}_M^2(x)$  with the appropriate quantile of  $\chi^2(k)$ , one can discard those calibration curves which are unacceptably far from the average, taking into account variances in the individual frequencies.

One of the problems with the above approach is that the computed mean can be influenced by presence of outliers in the training data and thus the resulting outlier detection may fail. Typical solution to this issue is to substitute the mean and standard deviation with the robust statistics such as the median and median absolute deviation, as proposed by, e.g., Hampel [2]. Nevertheless, our experiments have shown that such a modification does not bring required performance improvement in our task. The reason is that our datasets contain only small numbers of elements which are moreover fragmented into multiple subclusters.

As an answer to this problem, we propose a combined approach where the agglomerative hierarchical clustering is supported by the actual outlier detection in each divisive step. As for the outlier detection, we propose to estimate means and standard deviations for sets mutual distances between individual measured curves rather than directly for the curve data. This modification provides better notion about distribution of distances within and between clusters, where those within clusters tend to be significantly smaller than between clusters. In this context, we define the outlier as the curve having unusually large distance from the others within cluster compared to the rest of within-cluster distances.

As in our antenna calibration collection it is quite usual that clustered datasets contain 4+ curves, the robustness of the outlier detection method can be further improved by removing the most extreme value from the dataset before determining the outlying observation. When the largest distance is removed from calculation of the mean and standard deviation, the estimated confidence interval better reflects valid distances within the cluster and thus the outlier is more likely to be recognized. In the limit case, when the outlier has to be determined in a group of just 3 curves, we use a heuristics approach based on the comparison of two means, where the test statistic is defined by the two-sample  $t$  statistic:

$$t = \frac{\mu_1 - \mu_2}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}.$$

In this case, the two populations are represented by frequency-wise differences between two closest and two furthest curves of the three. We assume that the significant difference is observed only when the furthest observation is the outlier.

The step-by-step process of the proposed combined approach can be explained as follows. The first iteration starts by determination of two clusters in the input dataset using the agglomerative hierarchical clustering [1]. Then the mean and standard deviation is calculated for the pair of the shortest Euclidean distances within the clusters. The shortest between-cluster distance is then compared to the 98-th percentile of the Gaussian distribution given by the mean and standard deviation estimated in the previous step. If the distance does not exceed the threshold, the clusters are merged and the resulting cluster is returned. Otherwise, the algorithm repeated for each of the two clusters recursively. In the case of 3 analyzed curves, the outlier detection is performed using the heuristic approach described earlier. Using this procedure we are able to separate curves into distinct clusters automatically, without knowing the number of clusters in advance.

Finally, having all the clusters recognized correctly, utilizing the traditional  $\chi^2$ -based approach, we propose the following procedure for validating new calibration measurements:

1. estimate  $\mu$  and  $\sigma$  vectors for the curves in each cluster,
2. given the validated curve  $x$ , calculate  $\widehat{D}_M(x)$  for all the recognized clusters,
3. check if the  $\widehat{D}_M$  value for any of the cluster falls behind the required quantile of the  $\chi^2$  distribution; if yes, the tested curve can be considered as valid; if no, there is strong evidence that the curve is invalid.

### 3. Results

In order to evaluate performance of the proposed combined clustering approach in comparison with the traditional approach using the total cluster variance criterion, an exhaustive testing on 170 groups containing 1126 calibration curves was conducted. Beforehand, all the curves within each group were manually clustered by a field expert, which provided a baseline for evaluation of the automated methods. In Tab. 1., the final results are shown. One can clearly see that our clustering method delivers approx. 2x higher success rate (with respect to the expert opinion) than the traditional approach. In absolute values, the traditional method succeeded in recognizing 37.06% of clusters, while our method succeeded in 80.59% of the cases.

Table 1. Result of a test on 170 groups containing 1126 calibration curves for both considered automatic clustering methods.

Algorithm	Success rate w.r.t. expert opinion
Total cluster variance criterion	37.06%
Proposed combined clustering	80.59%

### 4. Conclusions

During our research we have created a unique database of antenna calibrations allowing for diverse analyses according to various criteria such as antenna model, serial number, calibration method or measurement parameters.

By developing a tailored clustering method utilizing the proposed outlier detection algorithm, we were able to separate antenna factor curves into subclusters without in advance knowledge of the number of clusters. Compared to the traditional automated clustering approach, our method provided 2x higher success rate reaching up to 80.59%.

Having all the clusters recognized correctly, we proposed procedure for validating new calibration measurements utilizing the traditional  $\chi^2$ -based approach to the model testing.

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