Accuracy of the Body Surface Potential Approximation

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Abstract. *The problem of construction of body surface potential maps (BSPM) is important for many aspects of clinical practice. This study compares several methods of BSPM approximation, including three variants of spherical harmonics approximation and two variants of Laplacian interpolation. The methods were evaluated for their accuracy using tomographic and electrophysiological studies, as well as the results of computer modeling. The accuracy measures were locally estimated giving the distribution of these measures over the whole body and their variation during the cardiac cycle. Recommendations on the method implementation and proposals for further research are formulated.*

Keywords: Body Surface Potential; Noninvasive Electrocardiographic Imaging; Electrocardiotopography; Inverse ECG Problem.

1. Introduction

A comprehensive knowledge of the cardiac electric field is gained from electrophysiological studies using multiple electrodes (up to 400) evenly distributed over the whole body surface. Diagnostic analysis of synchronously recorded cardiac electrical signals may be conducted in two ways.

Historically the first approach is to construct and analyze electrocardiotopograms, i.e., body surface potential maps (BSPM) [1]. The position of electrodes on such maps is conditional, as it reflects their relative positioning on the chest with no allowance for individual inter-patient variability of their coordinates. For graphic BSPM presentation, local 2D approximation (interpolation) is conducted for continuous smooth filling of gaps between conditional points of measurement. BSPM approximation (interpolation) on detailed polygonal models of the body surface was intensively investigated as a tool for consolidation of the diagnostic experience with the data from different multi-electrode lead systems (e.g., [2 – 4]).

Another approach to diagnostic analysis implies the solution of inverse ECG problem and reconstruction of electrophysiological processes in the myocardium. BSPM approximation on the numerical model of the body surface is usually the preliminary step of the inverse ECG problem solution.

BSPM approximation is also necessary for the detection and spatial filtration of noise and various signal distortions.

2. Subject and Methods

Various methods of BSPM approximation were evaluated for their accuracy using tomographic and electrophysiological studies, as well as the results of computer modeling.

Data Preparation

Torso and heart models represented by triangular meshes and coordinates of 240 electrodes were obtained from electrophysiological studies using Amycard diagnostic complex [5].

Torso and heart tetrahedral finite element presentations and other necessary data, including sites and electrical parameters of stimulation, parameters of bidomain myocardial model, electric conductivity of the medium, were used as input data for the Cardiac Chaste software complex [6]. As a result, we obtained heart surface electrograms and potentials and electrocardiograms on the body surface during one cardiac cycle in each vertex of the corresponding triangular meshes.

Let us introduce the following notation: $\boldsymbol{r}_i = (x_i, y_i, z_i)$, Cartesian coordinates of the *i*-th vertex of the triangular mesh; t_j, time point of the *j*-th ECG. Then, $u(\boldsymbol{r}_i, t_j) = u_i(t_j) = u_{ij}$ are potentials in the *i*-th vertex of the torso triangular mesh in the *j*-th time point; $u_i(t)$, ECG in the *i*-th vertex; $\mathbf{u} = \{u_{ij}\}$.

'Measured' ECGs were defined as ECGs recorded at the electrode sites, and all of the rest records were classified as 'unknown'. Each of the below listed methods was used to approximate ECGs over the whole torso surface.

Approximation Methods

Multipole expansion or Spherical harmonics approximation. The general solution of the Laplace equation for a bounded domain that does not contain a neighborhood of the origin, can be presented as an expansion in terms of the singular and regular solid spherical harmonics [7]. Then ECG in any torso point can be presented in the form:

$$u(\boldsymbol{r},t) = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} \left(a_n^m(t)\, S_n^m(\boldsymbol{r}) + b_n^m(t)\, R_n^m(\boldsymbol{r}) \right), \tag{1}$$

where $S_n^m(\boldsymbol{r}) = r^{-(n+1)} Y_n^m(\theta,\varphi)$, $R_n^m(\boldsymbol{r}) = r^n Y_n^m(\theta,\varphi)$ are solid spherical functions; $Y_n^m(\theta,\varphi)$ are real spherical functions or harmonics; n and m are degree and order of the spherical function; (r,θ,φ) are spherical coordinates of the point $\boldsymbol{r}$; $a_n^m(t), b_n^m(t)$ are the time-dependent expansion coefficients.

The three variants of the spherical harmonics subset were used for approximation: singular solid functions with origin in the torso centre (SSF-S-Tc); singular solid functions with origin in the heart centre (SSF-S-Hc); singular and regular solid functions in the heart centre (SSF-SR**).**

Laplacian interpolation (LI) [8]. The Laplacian **L** for the torso surface triangle mesh is calculated and the ECGs $u_i(t)$ in all "unmeasured" vertices of the surface are adjusted to globally minimize the two-norm of the mesh potential Laplacian $\|\mathbf{Lu}\|$. There are several reasons that Biharmonic interpolation (BI) gives more smooth interpolation. To realize it, the double Laplacian two-norm $\|\mathbf{LLu}\|$ should be globally minimized.

Accuracy Measures

The differences between the potentials and their approximations were documented by several measures along only time, along only space and altogether.

The measures along time are integral measures of error during the whole cardiac cycle for every vertex. They are maps. The three such accuracy measures were used

$$\text{MaxErrMap}(\boldsymbol{r}_i) = \max_j \left| u_{ij} - \hat{u}_{ij} \right|, \text{ maximal error in vertex } i\,; \tag{2}$$

$$\mathrm{RMSErrMap}(\boldsymbol{r}_i) = \sqrt{\tfrac{1}{J}\sum_{j=1}^{J}(u_{ij} - \hat{u}_{ij})^2}\ \text{, root mean square error in vertex } i; \qquad (3)$$

$$\mathrm{RelDifMap}(\boldsymbol{r}_i) = \sqrt{\sum_{j=1}^{J}(u_{ij} - \hat{u}_{ij})^2 \Big/ \sum_{j=1}^{J} u_{ij}^2}\ \text{, relative mean square error in vertex } i\,. \quad (4)$$

The measures along space are integral measures of error over the whole torso surface. They are functions of time. The three such accuracy measures were used: $\mathrm{MaxErrF}(t_j)$, maximal error at instant *j*; $\mathrm{RMSErrF}(t_j)$, root mean square error; $\mathrm{RelDifF}(t_j)$, relative error. Their formulae are quite the same as (2)-(4), if in max and sum operators change *j* for *i*.

3. Results

The BSPMs were obtained for the two variants of the torso and heart triangular meshes and 7 variants of the cardiac pacing sites as described in the "Data preparation" section above. The approximation methods were applied, and the accuracy measures were locally estimated giving the distribution of these measures over the whole body and theirs variation during the cardiac cycle. The results for one typical case of accuracy variation during the cardiac cycle are shown in fig. 1, 2.

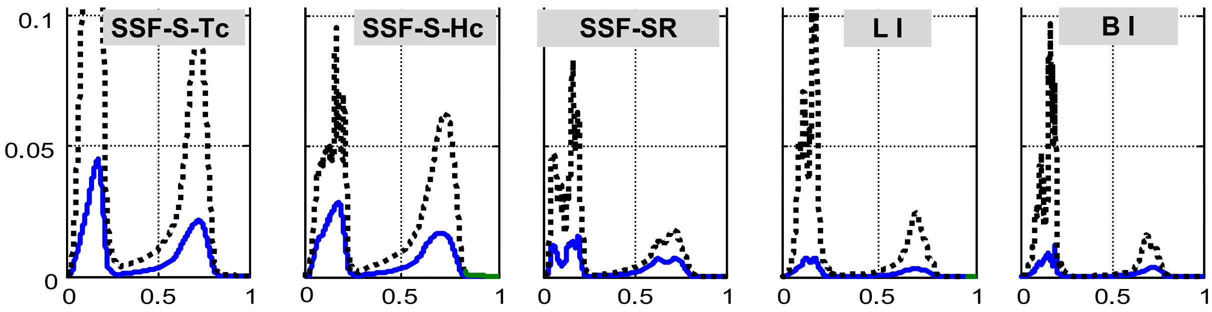

Fig. 1. Accuracy (RMSErrF, solid line; MaxErrF, dotted line) of the BSPM approximations during one cardiac cycle for different methods (designations of methods are introduced above).

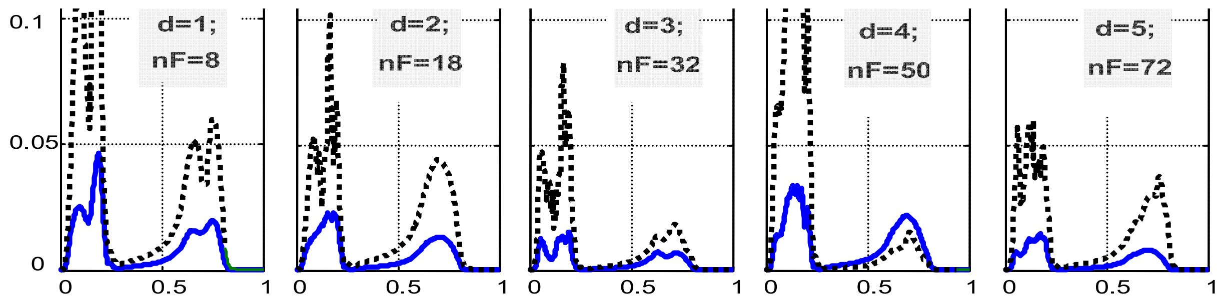

Fig. 2. Accuracy (RMSErrF, solid line; MaxErrF, dotted line) of the BSPM approximations during one cardiac cycle for the SSF-SR variant of the multipole expansion with different d, maximum degrees of spherical harmonics used for approximation (nF, total number of functions).

4. Discussion

No one methodology can be optimal in every context for every accuracy measure. Only three of the five methods tested may be advantageous in some cases or in terms of a certain accuracy measure. These include SSF-SR version of multipole expansion, Laplacian interpolation, and Biharmonic interpolation. It is more often that the latter turns out to be preferential. LI or BI always wins in terms of mean square metric, but SSF-SR or BI always superior in terms of uniform metric.

Multipole expansion with the origin in the torso center is always least accurate in terms of all measures. Multipole expansion should be as close as possible to the ‘center of gravity’ of the sources of the cardiac electric field.

Examining the maps of accuracy measures (2)-(4) reveals the regions with high errors. This is useful for the development of new and improved BSPM approximation methods.

It is noteworthy that accuracy of the BSPM approximation method was previously assessed using an *in vivo* model of the perfused canine heart placed into the electrolyte body model [4], whereas in the present study, we have conducted simulation experiments *in silico*.

The results obtained encourage a number of even more interesting studies, particularly, to evaluate the accuracy of the inverse ECG problem solution based on the potential reconstruction on the myocardial surface. It will be of interest to compare the results of such studies and to evaluate the effect of approximation accuracy on the accuracy of the inverse problem solution.

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