Absolute Flow Velocity Measurements by Means of the Thermal Waves

A. Rachalski, M. Bujalski, P. Ligęza, E. Poleszczyk

Strata Mechanics Research Institute, Polish Academy of Sciences, Krakow, Poland, Email: rachalsk@img-pan.krakow.pl

Abstract. The paper considers of applying the thermal wave method of absolute measurements of very low flow velocity. The method is based on approximate analytical solution of thermal wave propagation in flowing gas. Some results of air flow velocity measurements were presented.

Keywords: Flow Velocity Measurement, Thermal Wave, Absolute Measurement.

1. Introduction

The idea of flow velocity measurement by means of thermal wave method is not new. The method consists of measuring the time of wave passage within the flow at known distance. Since early works of Kovasznay [1] and Walker and Westenberg [2] the method has been developed, and nowadays is used in flowmeters widely applied in manifold areas of experimental sciences and technology. The interesting application of the method, intensively progressing recently is measurements of micro and nanoflows [3],[4]. The theoretical analysis of the method is commonly restricted to particular solutions of measuring devices and circumstances of measurement. The thermal wave propagation is sensitive to flowing gas parameters, therefore measuring devices need to be calibrated. Kiełbasa presented the analytical solution of thermal wave propagation in a flowing gas, and showed the necessary conditions of absolute measurements the flow velocity [5], a shortly described by Rachalski [6]. To say briefly, the absolute method needs the conditions when the thermal diffusion does not affect the thermal wave velocity. In air, under laboratory conditions the lover limit of flow velocity is about 30cm/s.

2. Theoretical model

The governing equation of thermal wave propagation is advection-convection equation:

$$\frac{\partial T}{\partial t} = \operatorname{div}\left(\kappa \operatorname{grad} T\right) - V_G \frac{\partial T}{\partial x} + \frac{Q(t)}{\rho c}$$
(1)

where

Q(t) intensity of wave source

T gas temperature

 V_G flow velocity

 κ thermal diffusivity

 Δx distance between detectors

 ω angular frequency of thermal wave

For sinusoidal wave and the probe orientation presented on Fig.1 Kiełbasa [5] came up to the expression for phase shift:

$$\Delta\varphi(\Delta x,\omega,\kappa,V_G) = \frac{V_G \Delta x}{2\kappa} \sqrt{\frac{1}{2} \left(\sqrt{1 + \frac{16\kappa^2 \omega^2}{V_G^4}} - 1\right)}$$
(2)

From Eq.3 we can derive the condition of absolute measurement of flow velocity; if the fraction in the inner square root is enough small (but not equals to zero), i.e.:

$$\frac{4\kappa\omega}{V_G^2} \Box \quad 1, \tag{3}$$

after expanding the root into series we obtain approximate relationship:

$$V_{T} = \frac{\omega \Delta x}{\Delta \varphi} = V_{G} \tag{4}$$

From Eq.2 and Eq.4 we obtain the relationship between gas flow velocity V_G and thermal wave velocity V_T :

$$V_{T} = V_{G} \sqrt{\frac{1}{2} \left(\sqrt{1 + \frac{16\kappa^{2}\omega^{2}}{V_{G}^{4}} + 1} \right)} .$$
 (5)

Equation 3 shows dispersion of the thermal wave, which may be significant in low flow velocity range. Since the thermal diffusion



Fig. 1. Probe orientation in flowing gas.

in small flow velocities cannot be omitted, the idea is to determine flow velocity V_G by solving the set of equations:

$$\Delta \varphi_i = \frac{V_G \Delta x}{2\kappa} \sqrt{\frac{1}{2} \left(\sqrt{1 + \frac{16\kappa^2 \omega_i^2}{V_G^4} - 1} \right)}, \qquad (6)$$

where $\Delta \varphi_i$ is measured phase shift of wave of ω_i frequency. Even though Eq.6 contains only the two unknown variables V_G and κ we use, for better accuracy, set of more than two equations. Since of uncertainty of the phase measurement this set of equations is inconsistent, so it must be solved by fitting data points, accordingly to Eq.2 [7]. Above analysis concerns with sinusoidal wave, but may be applied to square waves. Convenient method of the phase shift determination is applying Fourier analysis to recorded detectors' signals.

3. Results

The research was performed in the wind tunnel in velocity within the range of 5 to 30 cm/s. For generating and recording thermal waves, digital anemometer- thermometer CCC2000 was used [8]. The both transmitter and detectors wire were made of tungsten 5μ m in diameter. The distance between detectors was 2mm, and between the first detector and transmitter 2mm. Orientation of the sensor in the flow is shown in Fig.1. Figure 2 presents a measured velocity



Fig.2. The measured sine thermal wave velocity vs. wave's frequency for flow velocity 7.5 cm/s and 12 cm/s.



Fig.3. The measured phase shift vs. wave's frequency for flow velocity 7.5 cm/s and 12 cm/s.

of sine thermal wave of various frequencies. For flow velocity of 7.5 cm/s the dispersion is more significant than for flow velocity 12 cm/s, accordingly to discussion presented above. The solid lines represent the numerical fit of Eq.5 to data points. The measured phase shift for sinusoidal wave is shown in Fig.3. The solid lines present a numerical solution of Eq.6 obtained by means of nonlinear regression. Rather than sinusoidal wave, a square wave is applied in thermal wave method, see e.g. [9]. Other idea is to exploit natural or artificial



Fig.4. Comparison of measured flow velocity for sinusoidal and square wave as well cross-correlation of pseudorandom signals.

fluctuations of the flowing gas temperature [10]. In Fig.4 presents a comparison of flow velocity measurements by means of spectral analysis of sine and square waves and by means of crosscorrelation of random wave. It is shown a good agreement between sinusoidal and square wave, while cross-correlation gives higher values of velocity. The all methods display lower values of velocity than established in the tunnel. In fact, that actually the flow velocity in the area between wave's detectors is being measured. This velocity, is smaller than the inflow velocity, because of the velocity is decreased in the wake of the wire.

4. Conclusions

To absolute measurement of very low flow velocity, the thermal wave methods needs the spectral analysis of the signal, since the thermal diffusion affects the wave velocity. To make the method practice, necessary is reducing the influence of the velocity wake on measurement results to an acceptable level.

Acknowledgements

This study was performed under the research project: "Investigation of spatial propagation and optimization of methods for generation, detection and analysis of temperature waves in the aspect of absolute measurement of flow velocity and thermal diffusivity of gases" financed by Polish National Science Centre upon a decision number DEC-2012/07/B/ST8/03041. Experimental research was realised in a wind tunnel financed by *Fundusz Nauki i Technologii Polskiej* (Fund of Polish Science and Technology), grant no. 682/FNiTP/34/2011.

References

- [1] Kovasznay LSG. Hot wire investigation of the behind cylinders at low Reynolds number. *Proceedings of the Royal Society London A* 198: 174-190, 1949.
- [2] Walker RE, Westenberg AA. Absolute low speed anemometer. *Review of Scientific Instruments* 27 (10): 844-848, 1956.
- [3] Shoji S, Esashi M. Microflow devices and systems. *Journal of Micromechanics and Microengineering*. 4(4):157, 1994.
- [4] Berthet H, Jundt J, Durivault J, Mercier B, Anglescu D. Time-of-flight thermal flowrate sensor for lab-on-chip applications. *Lab on a Chip* 11(2):181, 2011.
- [5] J. Kiełbasa. Fale cieplne w metrologii powolnych przepływów (*in polish*), Ed. AGH, Kraków, 1976.
- [6] Rachalski A. High-precision anemometer with thermal wave. *Review of Scientific Instruments*, 77 (9): 095107, 2006.
- [7] Rachalski A. Absolute measurement of low gas flow by means of the spectral analysis of the thermal wave. *Review of Scientific Instruments*, 84 (2): 025105, 2013.
- [8] Ligęza P. Four-point non-bridge constant-temperature anemometer circuit. *Experiments in Fluids* 29(5): 505-507, 2000.
- [9] Biernacki Z, Kurkowski M, Zloto T, Ptak P. Analysis of exploitation and metrological properties of a wave thermoanemometer system. *Measurement Science Review*, 1(1): 131-134, 2001.
- [10] Ligęza P. Use of Natural Fluctuations of Flow Parameters for Measurement of Velocity Vector. *IEEE Transactions on Instrumentation and Measurement*, 63 (3): 633-640, 2014.