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A-D CONVERSION WITH DITHER SIGNAL-POSSIBILITIES AND LIMITATIONS

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Abstract

AD conversion with dither signal may have the following aims: to improve reconstruction (for the purposes of quantization), to reduce non-linear distortion, to produce increased resolution of the operation of quantization, and to reduce the non-linearity errors of an actual analog-to-digital converter (ADC). These advantages are achieved at the price of limiting a number of characteristics of AD conversion systems which apply dither. There is also a relationship between the nature of the non-linearity of a function and the efficiency of its reduction by means of the application of dither.
The paper discusses the mechanism of the impact of AD conversion with dither signal on the characteristics of the operation of quantization and on certain properties of an actual ADC.

1. INTRODUCTION

In the course of the development of measuring systems with AD conversion, this operation has been more and more markedly associated with the input of the systems. In measuring systems of the current generation, AD conversion is preceded only by a prefatory analog-to-analog (AA) conversion. The measuring process consists of two stages: the acquisition (AA + AD), and the estimation, which is a digital-to-digital (DD) conversion. Measurement information is retrieved through a DD conversion of the acquired instantaneous values of the signal. As a matter of course, it is desirable that after the AD conversion, the discrete form of the measuring signal contain, insofar as this is possible, as much information about the measured quantity as the continuous form. Therefore, the precision of measurement significantly depends on the quality of this operation.

The characteristics of AD conversion systems may be modified by means of the hardware method, the software method and the signal method (with dither). The latter method, applying dither signal, produces effects which cannot be achieved by the other methods.

2. ESSENTIAL TERMS

Dither signal may be applied in order to modify either quantization understood as a general type of signal conversion or the characteristics of an actual ADC. To avoid ambiguity in the following considerations, certain terms which appear in them are defined:

- x - the input signal of a quantizer, signal undergoing quantization,
- x_q - the output signal of a quantizer, signal after quantization,
- $Q(x)$ - the characteristic of an ideal operation of quantization,
- $L(x)$ - a reference, the characteristic of a quantizer with an infinitely high resolution (the linear function of the signal x),
- Quantization error e - the difference of the signal after quantization (the analog equivalent of the digital representation) and the signal before quantization $e = x_q - x$,
- Error of the operation of quantization (error resulting from the non-linearity of the operation of quantization), $g(x)$ - the difference of the characteristic of the operation of quantization and the characteristic of a linear operation $g(x) = Q(x) - L(x)$.

3. THE AIMS OF THE APPLICATION OF AD CONVERSION WITH DITHER SIGNAL

3.1. The Reduction of the Effects Produced by the Characteristics of the Operation of Quantization

3.1.1. Improved Reconstruction

Quantizing is a non-linear operation which modifies the parameters of the signal undergoing conversion. The modification of the properties of the measurement signal produced by changing its form from continuous to discrete affects the estimation. The degree to which the signal, upon quantization, preserves the parameters of the signal undergoing conversion depends, among other factors, on whether the signal undergoing conversion qualifies for quantization with reconstruction.

Reconstruction is applied to the function of probability density, the moments and the power density spectrum of the signal undergoing quantization, based on, respectively, the function of probability density, the moments and the power density spectrum of the signal after quantization [1]. Signal reconstruction with quantization occurs if the characteristic function of the signal, Φ_x meets the requirements specified in Widrow's theory. In this event the quantization error has certain specific properties, being white noise with a rectangular function of probability density, not correlated with the signal undergoing quantization [7]. If Φ_x does not meets Widrow's requirements, quantization with dither signal is applied to modify this property [3]. The mechanism of the improvement of reconstruction by means of the application of dither signal consists of the replacement of the function Φ_x by the function $\Phi_{x+d} = \Phi_x \cdot \Phi_d$ [1]. This is done through quantizing the signal *x+d* rather than the signal *x*.

$$f_x \to f_{x+d} = f_x * f_d \xrightarrow{F} \Phi_x \cdot \Phi_d = \Phi_{x+d} \leftarrow \Phi_x \tag{1}$$

The dither signal is selected in such a manner that its function of probability density, f_d, is such that the function Φ_{x+d} meets Widrow's requirements, or that it meets them to a higher degree than the function Φ_x. A benefit of this replacement is improved reconstruction, and consequently a desirable modification of the characteristics of the quantization error (whitening or the absence of correlation with the signal undergoing quantization).

3.1.2. The Reduction of the Non-Linear Distortion of the Operation of Quantization

Reduced non-linear distortion is achieved through the application of the *randomizing effect* of dither signal on the *quantization error*. A case where it is necessary to reduce non-linear distortion is the quantizing of periodic signals. Additional lines appear in the spectrum of a signal after quantization, which have not been visible in the spectrum of the signal undergoing quantization. These result from the non-linear conversion of the signal. Here, the quantization error is coherent. Upon the application of a suitable dither signal, the quantization error changes from coherent to random, and does not appear as significant harmonic distortion. This amounts to a reduction of non-linear distortion. In this situation, random dither is the most effective. Pseudorandom signal is most usually applied. Pseudorandom dither is a waveform having the characteristics of noise but that can be accurately repeated [4].

3.1.3. Increased Resolution of the Operation of Quantization

In order to produce increased resolution or to reduce non-linear distortion in a system having the nominal characteristic of quantization $Q\ (x)$ and the error of the operation of quantization $g\ (x)$, quantization with dither signal and the averaging of the signal upon quantization are required. This mode of conversion is identical to quantization in a system having the nominal characteristic of quantization $\overline{Q}(x)$ and the error of the operation of quantization $\overline{g}(x)$. Assuming that the parity of the function f_d is even, in this case the following situation is produced [5,6]

$$\overline{Q}(x) = Q(x) * f_d(x) \tag{2}$$

$$\overline{g}(x) = g(x) * f_d(x) \tag{3}$$

and

$$\bar{g}(x) = \bar{Q}(x) - L(x) \tag{4}$$

Increased resolution is achieved through the *attenuating effect* of dither signal on *the error of the operation of quantization*. It is so because $\bar{g} < g$. Accordingly, information on the varying value of the signal inside the range of quantization may be retrieved. If the dither signal is probabilistic, then functions Q and g are convoluted with the function of the probability density of dither signal f_d. If, however, the dither signal is deterministic, then the weighted average of functions Q and g is calculated based on the function of the distribution of the values of f_d. Regardless of the case, the function f_d is selected in such a manner that the magnitude of g is suitably decreased.

Increased resolution amounts to improved linearity of the operation of quantization.

3.2. The Reduction of the Non-Linearity Errors of an Actual ADC

3.2.1. The Differential Non-Linearity Error

The differential non-linearity error (DNL) may be represented as a function which is the difference of the actual characteristic, Q_R, and the ideal characteristic of the operation of quantization: $DNL = Q_R(x) - Q(x)$. This is a sequence of rectangular pulses of the magnitude $\pm q$ and the variable width $r_k - l_k$ which width depends on the value of the error for the code in question, and having its axes of symmetry in points x_i.

$$DNL(x) = q \sum_{i=1}^{N} \alpha_i \cdot \Pi\left(\frac{x - x_i}{r_i - l_i}\right) \qquad \alpha_i \in \{-1, 0, 1\} \qquad x_i = l_i + \frac{r_i - l_i}{2} \tag{5}$$

The DNL error is reduced by means of quantization with dither signal and the averaging of the signal upon quantization. This mode of conversion is identical to quantization in a system having the differential non-linearity error $\overline{DNL}$

$$\overline{DNL}(x) = q \sum_{i=1}^{N} \alpha_i \cdot \Pi\left(\frac{x - x_i}{r_i - l_i}\right) * f_d(x) \tag{6}$$

The DNL error is reduced through its redistribution over a wider range and a simultaneous smoothing, which decreases its magnitude [2]. Due to the delocalization and randomization of the DNL error, the errors for all codes become similar, or are integrated, and cease to be regular. The effects of the reduction depend on the nature of the function f_d to a low degree only. The value of peak-to-peak dither is more important here. In each case there is a limit to this value, above which adjacent errors are entirely integrated and further improvement ceases to be significant.

3.2.2. The Integral Non-Linearity

Unlike the DNL error, the integral non-linearity error (INL) is non-local. Due to the nature of this non-linearity, AD conversion with dither signal does not effectively reduce it. A more efficient method involves the application of algorithms which correct the results of quantization based on information on the INL error acquired at the stage of the calibration of the AD conversion system.

Furthermore, the INL error is a considerable hindrance to increasing resolution, or increasing the effective number of bits, by means of a method applying dither.

4. Modifying the Characteristics of AD Conversion Systems Applying Dither Signal

Quantization with dither signal does not unconditionally refine the process of AD conversion. The application of dither signal reduces the following quantities: signal-to-noise ratio of the ADC output, dynamic range, conversion rate, input signal bandwidth, sampling rate.

Improvements that are achieved in the area of resolution or linearity are at the sacrifice of losses in effective bandwidth and response time.

5. The relationship between the nature of the non-linearity of a function and the efficiency of its reduction by means of the application of dither

A common quality of the characteristic of the operation of quantization, Q, the error of the operation of quantization, g, and the DNL error of an actual ADC is that they all are discontinuous non-linear functions. Conversely, the INL error is a smooth non-linear function.

Therefore, AD conversion with dither signal and the averaging of the signal upon quantization replaces a conversion described by the functions Q, g and DNL by one described by the functions, $\overline{Q}$ $\overline{g}$ and $\overline{DNL}$. The functions Q, g and DNL are smoothed, and the magnitudes of the latter two are decreased.

A convolution of a non-linear discontinuous function with a dither signal having an appropriately selected function f_d turns the non-linear discontinuous function into a non-linear continuous function. Thus, quantization with dither signal has particularly substantial impact on quantities described by non-linear discontinuous functions or functions whose values vary in a relevantly non-linear manner. On the other hand, it is difficult to produce a desirable effect of quantization with dither signal on the function of INL, which is smooth and non-linear, i.e., whose values hardly vary in a non-linear way. This is a case similar to the filtration of a signal which occupies the band (0, f_B) in a low-pass filter whose cut-off frequency is f_F. If $f_B < f_F$, then filtration does not occur. Conversely, if $f_B > f_F$, then filtration smoothes the signal through limiting the dynamics of its variation. Whether filtration occurs or not, depends on the dynamics of signal variation. Dither signal may affect the process or not, depending on the scale of the non-linearity of the variation of the values of the function. The more non-linear the variation, the more effective the dither signal method is.

6. Conclusion

AD conversion with dither signal may have the following aims:

- to modify the probabilistic model of the operation of quantization, in order to improve reconstruction for the purposes of quantization and to reduce non-linear distortion (randomizing effect on the quantization error),
- to modify the characteristics of the operation of quantization, in order to produce increased resolution, i.e., to improve the linearity (attenuating effect on the error of the operation of quantization),
- to reduce the DNL error of an actual ADC.

One should differentiate quantization error from error of the operation of quantization.

AD conversion with dither signal is an efficient method of reducing the effect of quantities described by discontinuous non-linear functions.

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