Minimisation of Measurement Errors and Optimisation with Correcting Procedures

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Abstract

At first it is shown how measuring errors – both dynamic and statistical ones – are depending of correcting procedures today realised without additional hardware using instrument computers. This leads to the well-known classical optimal filters with minimal total errors. Then the limitations due to parameter variations as well as to systems containing allpasses were discussed.
In the second part another optimal criterion given by the information theory is used leading to new filters: The information flow should be a maximum. Now the optimum is shifted to higher values of the degree of correction than in the first case.

1. Introduction

Today measuring instruments normally contain an instrument computer to realise a suitable processing of the measured data. In these cases it is possible without additional hardware to realise an algorithm for minimising the total error consisting of the two components: The dynamic and the statistical i.e. noise error. At first these problems will be treated and then another new optimisation criterion will be applied leading to information-optimal systems.

2. Classical optimal system

As well-known from then theory of optimal filtering the classical optimisation criterion "minimising the mean-square error" is used.
To show the main ideas as well as to prepare the new results in the next chapter a special case with great importance to practice may be treated: The behaviour of a measuring system should be corrected by a series-connected computer. If for instance the system is of first order

$$G_1(j\omega) = c_1 / (1 + j\omega T_1) \qquad (1)$$

the ideal frequency response of the correcting system should be [1;4]

$$G_{2id}(j\omega) = c_2 \ (1 + j\omega T_1) \qquad (2)$$

This frequency response cannot be realised. Therefore the frequency response has to be

$$G_2 (j\omega) = c_2 \ (1 + j\omega T_1) / (1 + j\omega T_2) \qquad (3)$$

leading to the corrected system

$$G_{ges} = G_1 G_2 = c_1 c_2 / (1 + j\omega T_2) \qquad (4)$$

Comparing (4) with (1) one learns that the new system now has a new time constant T_2 and limiting frequency $\omega_{c2} = 1/T_2$. We introduce the factor a describing the improvement of the dynamic behaviour of the corrected system

$$a = T_1 / T_2 \qquad (5)$$

Figure 1 shows both the step-answer function of an original and a corrected temperature sensor as an example with great practical importance of a system of first order.
Dealing with the two components of the mean-square error ε^2 - the dynamic error ρ^2 and the statistical or noise error P_z - as a function of the degree of correction a we get the results of Figure 2: The decreasing dynamic error has to be paid by an increasing statistical or noise error. This fact can be realised by comparing the Figures 1a with 1b, because in 1b the line is due to the greater noise not so clear than in 1a.

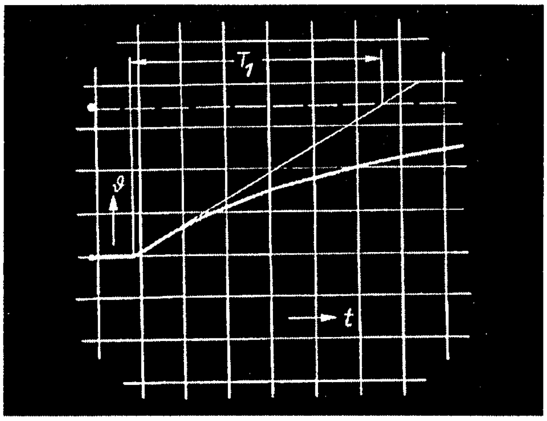

Figure 1. a) step-answer (transient) function of an uncorrected (original) temperature sensor

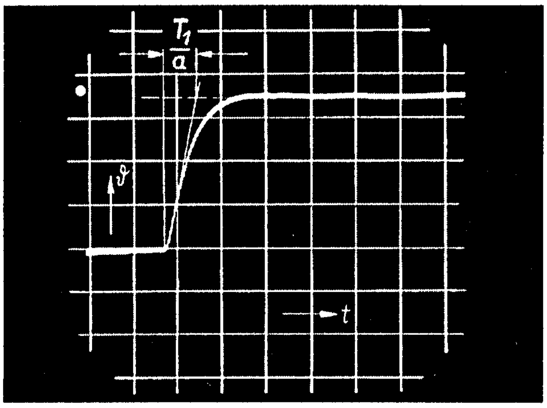

Figure 1 b) transient function of the corrected system, a = 10

The total error ε^2 is given by the sum of the two error components and leads to a minimum as shown in Figure 2. This means the well-known optimal filtering.

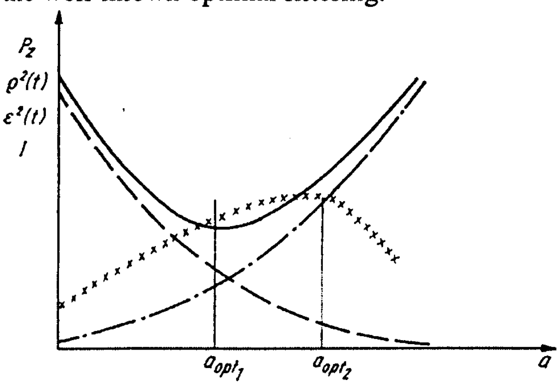

Figure 2. Errors and information flow as a function of the degree of correction
- - - dynamic error, -.-.- noise error,
---- total error, x x x x information flow

Now the question arises if it is still necessary to construct an original system with high quality because it is today possible using the instrument computer to improve the behaviour. Figure 3 shows the results of investigations: The better the quality of the original system the more efficient the correction!

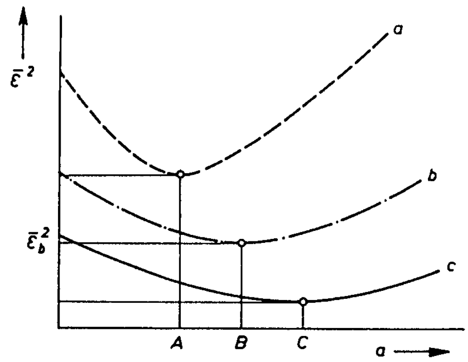

Figure 3. Errors of three systems of different quality as in Figure 2:
-------- High-quality original system
.--.--.-- Medium-quality original system
- - - - - Bad-quality original system

Last not least another effect should be mentioned limiting the degree of correction possible: To get an optimal correction the two time-constants T_1 in the equations (1) and (3) must be exact of the same value. In practice due to the parameter variations of the systems this is not possible. In this case the transient response will have a prolonged trail as shown in Figure 4 and so the advantage of the correction will be annihilated. This effect is called "parameter sensitivity".

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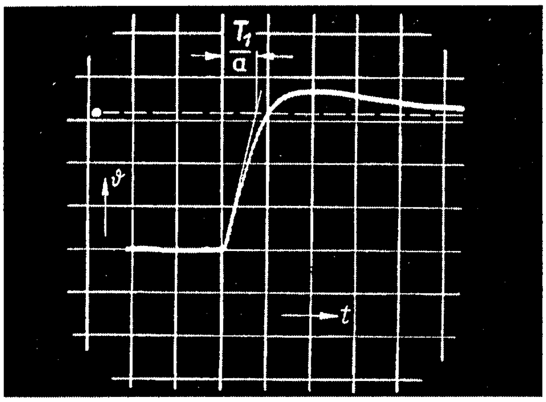

Figure 4. Influence of a difference between the time constants of the original and the correcting system $\Delta T = +0.2$ (parameter sensitivity)

Last it may be pointed out that it is not possible to correct systems containing allpasses without an additional time delay [4].

3. A new concept: Optimal information flow

As an introduction Shannon`s well-known channel capacity C_t will be explained [6]: With the signal-to-noise ratio P_s / P_n and the number of symbols per second $n = 2f_c = 1/t_r$ - where f_c is the critical frequency and t_r the response time, - the number of bits per second, the channel capacity is under the supposition of optimal coding

$$C_t = f_c \, \mathrm{lb} \, (P_s / P_n + 1) \qquad (6)$$

An intuitive approach explains the number under the binary logarithm to be the number of distinguishable power steps and the square root the number of amplitude steps [2;3;4;5]. Now we are using the same method to define an information flow I

$$I = f_c \, \mathrm{lb} \, (P_s / \varepsilon^2 + 1) \qquad (7)$$

Here also the system behavior should be corrected by means of a series-connected network or computer. Instead of minimizing the total mean-square error ε^2 - as shown before - now an optimal information flow I should be realized. Figure 2 shows as well the course of the error components and the total error as the information flow as a function of the degree of correction $a = f_c / f_o$ with the critical frequency of the corrected system f_c and of the original system f_o due to equation (5). The investigations show that because of the factor f_c in equation (7) the minimal error does not lay at the same value of the degree of correction a_{opt1} as with optimal filtering but at higher values a_{opt2}. Furthermore it may be emphasized that in this case also - as with optimal filtering - the advantages must be paid by increasing parameter sensitivity [3;4;5].

4. Conclusions

The investigations show that today by means of software using instrument computers it is possible to correct the behaviour of measuring systems. In the first part the principles as well as the limitations due to parameter variations and allpass-systems were demonstrated. Practical results of temperature sensors are given.

These methods leading to the so-called optimal filters with the criterion of a minimum total mean-square error then were expanded using the criterion of channel capacity or information flow. With an example it is shown that now the optimum is shifted to higher degrees of correction.

5. References

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