Signal Prediction Based On a Chaotic Attractor Model of the Electroencephalogram

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Abstract

This paper presents results of a non-linear study of the human electroencephalogram to establish the feasibility of extracting non-stationary information associated with internal or external events and stimuli. By invoking chaotic time series analysis techniques, short-term predictions are made on the attractor. Comparisons with the real evolution of the EEG could in principle yield stimulus-related information.

Introduction

Modern non-linear dynamic studies of different biosignals suggest [1]-[3], that in many cases measured signals can show attributes more closely matched to a low-dimensional chaotic attractor than that of random (infinite dimensional) noise. The long-term evolution of chaos is not predictable since information at one time does not correlate well with information at another distant time, despite the fact that the dynamics may be perfectly deterministic. Prediction in the short-term, however, can be quite accurate (up to a limit determined by the largest Lyapunov exponent). In an effort to explore the feasibility of harnessing prediction to extract information, we apply advanced techniques to test the concept on the EEG. We outline the algorithms employed by way of example on a system whose dynamics are known and present the results in the subsequent section.

Methods

To illustrate the theory we present a chaotic system which is a model of a Faraday disk driven self-exciting homopolar dynamo occurring in magnetohydrodynamic processes arising from a theory of the Earth's magnetic field [6]. The equations are:

$$\begin{aligned} \dot{x} &= x(y-1) - bzf(z) \\ \dot{y} &= a(1-x^2) - ky \\ \dot{z} &= xf(x) - mz \end{aligned} \qquad (1)$$

where

$$f(x) = 1 - e + esx$$

The values *a=20, b=2, k=1, m=1.2, s=1, e=0.1* exhibit chaos and *e=1* demonstrates asymptotic stability [7].

In studying many physical (deterministic) systems we do not have direct access to the state variables determining its evolution in *n*-dimensional space. By analysing an appropriate single channel of data we have access to what can be considered a 1-d projection of the original space. By invoking the *embedding theorem* [4], we can identify a space formally equivalent to the original made out of time delays of the observed variables:

$$\vec{Y}[n] = \{x[n], x[n+\tau_1], x[n+\tau_2]...\} \qquad (2)$$

where *x[n]* is the observed signal. Important technical questions arising in the application of this procedure are the choice of the delays and the number of coordinates required to accurately describe the dynamics.

Time Delay
In theory, if we have access to an infinite amount of infinitely accurate data any time delay will suffice [5]. Typically, of course, we only have access to a finite amount of data sampled at finite intervals. A good criterion in practice [5] is to take the first minimum in average mutual information versus time delay defined by

$$I(\tau) = \sum_{y[n],y[n+\tau]} P(y[n], y[n+\tau]) \log_2 \left[\frac{P(y[n], y[n+\tau])}{P(y[n])P(y[n+\tau])} \right] \tag{3}$$

Application of equation 3 finds the first minimum for x of equations 1 at $\tau = 23\tau_s$ where τ_s is the sampling time.

Embedding Dimension
A dimension $d>2d_A$ (where d_A is dimension of the attractor) is sufficient to always unfold an attractor when dealing with time lagged coordinates [5]. In many circumstances a smaller dimension d can be found. We use *false nearest neighbours* [5], which is based upon the concept that a pair of near points constituting false neighbours are close together as a result of projection on a dimension that is too small to properly unfold the attractor. Projecting to larger embedding dimensions will facilitate the elimination of false neighbours. The minimum dimension where all false neighbours are eliminated is called the *embedding dimension* d_E. The difference in distance R between nearest neighbours in dimension $d+1$ as compared to dimension d can be written as

$$\left(\frac{R_{d+1}^2[n] - R_d^2[n]}{R_d^2[n]} \right)^{1/2} = \frac{\left| x[n+d.\tau] - \hat{x}[k+d.\tau] \right|}{R_d[n]} \tag{4}$$

When this value is greater than a certain threshold, we consider the nearest neighbours to be false neighbours. This method suggests a dimension of 3 for unfolding the attractor for x in equations 1.

Prediction
If the source of a chaotic signal is stationary, the resulting attractor is invariant for any initial condition. We can exploit this fact by generating a database of local maps describing the flow from one neighbourhood to another (subject to the requirement that the attractor has been sufficiently sampled). To predict forward in time, a newly observed point's nearest neighbour in the data is sought and the corresponding local map is used to project it forward in time from that neighbourhood. This procedure can be performed iteratively to produce predictions of K steps ahead. Of course a positive largest Lyapunov exponent dictates that errors grow exponentially and ultimately limit the prediction horizon. Given a vector Y on the attractor we write its local linear map F as

$$\vec{Y}[n+1] = F_n(\vec{Y}[n]) = A_n + B_n\vec{Y}[n] \tag{5}$$

Selecting N nearest neighbours of Y, we can calculate the coefficients A_n and B_n by a least-squares method that minimises

$$\sum_{i=1}^{N}\left|\vec{Y}^{i}[k+1]-(A_n+B_n\vec{Y}^{i}[k])\right|^{2}$$

Figure 1a illustrates prediction for the system of equations 1, which is extremely accurate up to a point where the predicted and actual points suddenly diverge. Fig 1b demonstrates the correlation coefficient between predicted values and actual values at the point where the two start to diverge.

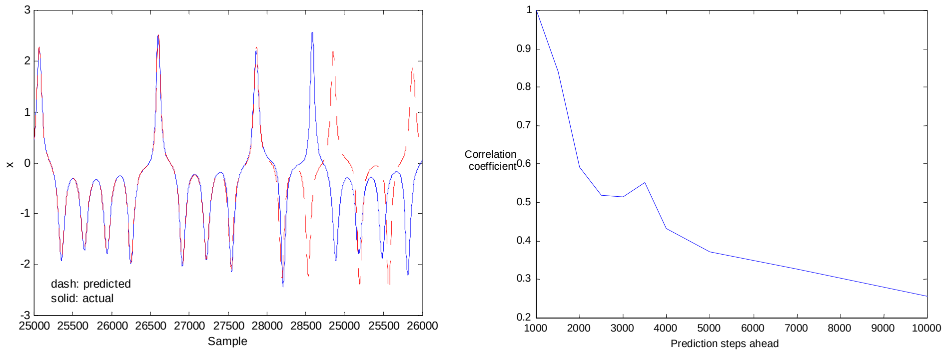

Fig 1a,b: Prediction and corresponding correlation coefficient plot

Results

Subjects were placed in a sitting position in a dimly lit, lead-shielded room. A single channel of EEG was acquired. Ag/AgCl 10mm electrodes were placed on O_z, and F_z, and the ground lead coupled to both ear lobes (this is the typical configuration used for extracting visual evoked potentials). Data was sampled at a rate of 250Hz from an amplifier with a gain of 50,000 and collected for a period of 3 minutes and low passed filtered with a cutoff frequency at 40Hz. A fourth order FIR filter was used since IIR filters can influence an effect on the measured nonlinear properties [5]. Referring to figure 2a,b we see that a time lag of 10 is appropriate and an embedding dimension of 8 is sufficient. The largest Lyapunov exponent for this data was estimated to be 0.35.

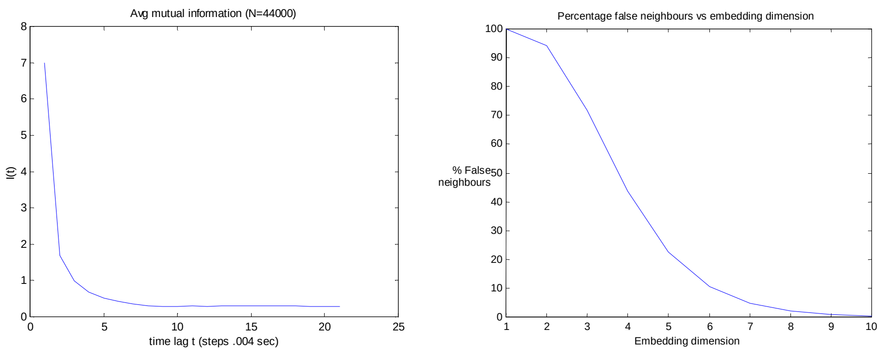

Fig 2a,b: Average mutual information and embedding dimension

To maximise prediction accuracy, we have found it wise to over-sample the data thus facilitating accurate local maps to be generate. Over-sampling data can lead to anomalies when calculating false nearest neighbours: Two successively sampled points close in time will always manifest themselves as near neighbours thus changing the global percentage of false nearest neighbours versus embedding dimension. To circumvent this, we operate on a decimated version of the data when calculating the false nearest neighbours above. Figure 3a,b illustrates some successful attempts at prediction. The largest global Lyapunov exponent indicates a limit of predictability in the long term but local prediction maybe better or worse than this [5].

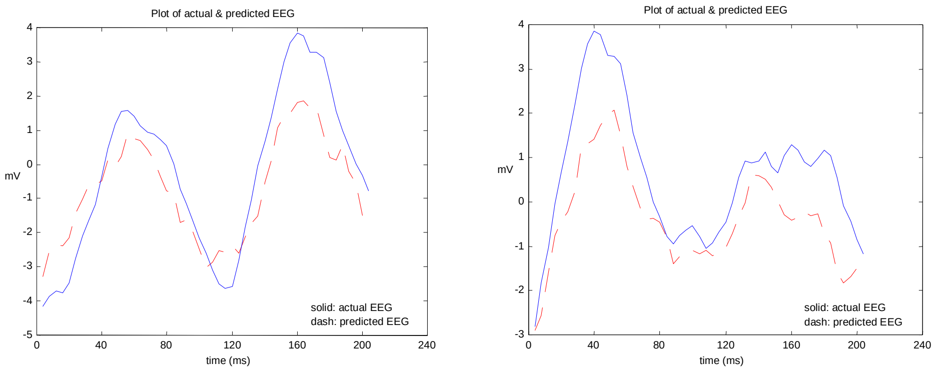

Fig 3a,b: Example of predictions

Discussion

We have attempted to predict measured brain electrical activity based on the assumption that the activity can be modelled by a chaotic attractor. Similar approaches could be made to other biosignals exhibiting chaotic attributes. If the future behaviour of a signal is known, responses occurring as a result of stimuli external to the system under study can be extracted. Although short term predictions are dependent on local Lyapunov exponents, the assumption of stationarity, and low measurement noise, the results are quite promising. In the context of the EEG, a well sought after goal is to extract single-trial evoked or event-related potentials. Unfortunately in this case, the amplitude ratio of the evoked potentials relative to the background activity is too low [8] for our method to work, however the authors feel hopeful that it could have some utility in studying other biosignals where the stimulus response could be quite large.

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